Optimization of a particle filter in case Track-Before-Detect

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Résumé – Dans cet article, nous nous intéressons à la détection et au pistage d'une cible en contexte Track-Before-Detect (TBD). Nous proposons ici un filtre particulaire efficace, fondé sur le choix d'une loi instrumentale pertinente motivée par des considérations de détection radar et permettant un gain significatif par rapport aux lois classiquement utilisées dans la littérature, notamment en terme de rapidité de convergence du filtre pour la détection. Nous déterminons également un nombre minimal de particules requis pour garantir des performances de détection intéressantes.

Abstract – In this paper, we are concerned with the detection and tracking of a target in TBD context. We propose here an efficient particle filter based on a relevant proposal density justified by radar detection considerations. This filter performs well compared to the classical laws used in the literature, especially in terms of speed of convergence for detection. We also identify a minimum number of particles required to ensure interesting detection performance.

1 Introduction

Tracking can be defined as the sequential estimation of the state of one or more potential targets based on noisy observations. In the classical target tracking methods, the measurements are typically the output of a processing chain, which consists of a matched filtering and a thresholding to isolate detection plots while maintaining a constant false alarm rate. This processing, which allows a considerable reduction in the amount of data to be processed during the tracking step, obviously results in a loss of information. It is of no consequence at high signal-to-noise ratio (SNR). By cons, at low SNR, a compromise must be struck between a high detection threshold, limiting the number of false alarms but also the probability of detection, and a lower detection threshold leading to a significant number of false alarms and thus increasing the complexity of the problem of association Plots / tracks.

Track-Before-Detect [2] approach proposes to overcome this predetection step and thus based the tracking on the raw measurements instead of plots. In the TBD approach, the detection is performed at the end of the processing chain, i.e. when all information has been gathered and integrated over time. This way information on the estimated state of a possible target is used in the detection problem. It is expected to get interesting performance, especially for weak targets. The classical tracking strategy must be redesigned. Indeed, the solutions of the Kalman family are here inapplicable because of the nature of observations (matrix type) and because of high non-linearities and possibly non-Gaussianity between the observations and the target's state. The first methods proposed, based on a criterion of Maximum Likelihood solved by dynamic programming [3] or by Hough transform [4], are unsatisfactory due to a running by block and a need to discretize the state space. Sequential solutions have been proposed recently [5, 6]. This kind of filter captures well complex observations and it copes well with non-linear and non-Gaussian systems,using a particles cloud propagating into the state space.

Apart from the problem of filtering a signal the second problem which also arises, is the detection problem in TBD. The strategy usually adopted to solve this problem is to introduce an additional binary state variable modeling the presence or absence of the target. In add, assuming an unknown reflected target power, the dynamical model of the system is augmented such that also estimation of the target power will be provided by the particle filter. These both choices induce an increase in the state space, and therefore the need to use a larger number of particles to get acceptable performance. It should be noted that the instrumental density used for the generation and for the propagation of particles must be adapted to the specific problem.

In this paper, we proposed a particle algorithm optimized for detection and tracking of a target in TBD context. The innovations consist to choose a relevant proposal density to sample the target state and its amplitude, and to identify a minimum number of particles required to ensure interesting detection performance. The proposal density introduced in this paper can be factorized between the target state vector, the associated binary variable and its amplitude, to sample the target state vector conditionally to the binary variable and the current observations. The integration over time of all information provided by mea-

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surements is carried through classical radar strategy, and led to spread smartly the particles in the state space. The minimum particles number required ensures to initialize at least one particle in the target vicinity with a probability fixed in advance.

The paper is organized as follow. In Section 2, we describe the state space formulation of the single target tracking approach using radar. In Section 3, we propose a particle filter based on a relevant proposal density and designed for a TBD application. We evaluate the performance of our algorithm in Section 4; a TBD application have been implemented and extensively simulated with an a priori unknown target power.

2 Problem formulation

2.1 System dynamic

We assume in this paper the target motion to be rectilinear uniform. We assume a discrete state model with a constant update time T. The state vector ($\mathbf{X}_k$) at step k is written as : $\mathbf{X}_k = [\mathbf{x}_k, \rho_k, s_k]^T$ where $\mathbf{x}_k = [x_k, \dot{x_k}, y_k, \dot{y_k}]$ represents the target's position and velocity in each of the cartesian coordinates, ρ_k is the unknown modulus of the target's complex amplitude and s_k is a binary variable representing the presence ($s_k = 1$) or the absence ($s_k = 0$) of the target. The evolution over time of s_k is described by a jump Markov process, whose transition probabilities $P_n = P(s_k = 1 \mid s_{k-1} = 0)$ and $P_m = P(s_k = 0 \mid s_{k-1} = 1)$ denote the probabilities of target birth and death respectively [8]. $\mathbf{x}_k$ is defined only for $s_k = 1$. In this case, if $s_{k-1} = 1$ the state's evolution is modeled by the linear equation:

$$[\mathbf{x}_k, \rho_k]^T = F[\mathbf{x}_{k-1}, \rho_{k-1}]^T + \mathbf{v}_k. \quad (1)$$

where the process noise $\mathbf{v}_k$ is assumed to be Gaussian with covariance matrix Q [8] and F the transition matrix defined by:

$$Q = \begin{bmatrix} Q_s & 0 & 0 \\ 0 & Q_s & 0 \\ 0 & 0 & q_i T \end{bmatrix}, F = \begin{bmatrix} F_S & 0 & 0 \\ 0 & F_S & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{avec,}$$
$$Q_S = q_s \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix} \text{ et } F_S = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}. \quad (2)$$

Else if $s_{k-1} = 0$, the position, velocity and ρ_k are respectively uniformly distributed over the observation window of the radar, the block $[v_{min}, v_{max}]^2$ and the interval $[\rho_{min}, \rho_{max}]$.

2.2 Measurement model

At step k, the radar system collects a set of noisy observations $\mathbf{r}_k$, which is assumed to be a 2-dimensional image consisting of $N = N_d \times N_\theta$ range-bearing cells. Each resolution cell (l, m) is centered in $(d_{min} + (l - \frac{1}{2})\Delta_d, \theta_{min} + (m - \frac{1}{2})\Delta_\theta)$ where Δ_d and Δ_θ represent range and bearing resolutions associated to the radar respectively ; d_{min} et θ_{min} are respectively the minimum range and minimum bearing reported by the radar.

Let's consider r_k^{lm} the signal measured in the range-bearing cell (l, m), described by the following nonlinear equation [6]:

$$r_k^{lm} = s_k \rho_k e^{i\varphi_k} h^{lm}(\mathbf{x}_k) + n_k^{lm}, \quad (3)$$

with $\varphi_k \in (0, 2\pi)$ and $h^{lm}(\mathbf{x}_k)$ is the received signal at the output of the matched filter in the cell (l, m) for a target characterized by the state $\mathbf{x}_k$. n_k^{lm} are independent samples of a complex white Gaussian noise of variance $2\sigma^2$ (*i.e.* the real and imaginary components are assumed to be independent, zero-mean white Gaussian with same variance σ^2). In practice, the range and bearing processing are performed separately. The received signal can then be factorized as $h^{lm}(\mathbf{x}_k) = h_d^{lm}(\mathbf{x}_k) \cdot h_\theta^{lm}(\mathbf{x}_k)$, where $h_d^{lm}(.)$ and $h_\theta^{lm}(.)$ denote respectively the signals at the output of matched filters in range and in bearing. Let's consider a elementary pulsed radar which transmits a linear frequency modulated signal ("*chirp*"), with bandwidth B and pulse duration T_e. $h_d^{lm}(.)$ then is written as [1] :

$$h_d^{lm}(\mathbf{x}_k) = \mathbb{1}_{|\tau^l| \le T_e} \left| \left(1 - \frac{|\tau^l|}{T_e}\right) \frac{\sin\left(\pi B \tau^l \left(1 - \frac{|\tau^l|}{T_e}\right)\right)}{\pi B \tau^l \left(1 - \frac{|\tau^l|}{T_e}\right)} \right|, \quad (4)$$

with $\tau^l = \left[d_k - \left(d_{min} + (l - \frac{1}{2})\Delta_d\right)\right] \frac{2}{c}$, c the speed of the electromagnetic wave and $d_k = \sqrt{x_k^2 + y_k^2}$, the radial range radar-target. Moreover, we assume here a phased array of N_a antennas ; the output $h_\theta^{lm}(.)$ satisfies the relationship [7]:

$$h_\theta^{lm}(\mathbf{x}_k) = \frac{\sin\left(\frac{N_a \Phi^m}{2}\right)}{N_a \sin\left(\frac{\Phi^m}{2}\right)}, \quad (5)$$

with $\Phi^m = \frac{2\pi d_a}{\lambda_P}\left[\cos(\theta_k) - \cos\left(\theta_{min} + (m - \frac{1}{2})\Delta_\theta\right)\right]$, d_a The distance between two row antennas, λ_P the wavelength related to the carrier frequency and $\theta_k = \arctan(\frac{y_k}{x_k})$, the target's bearing.

We define the Signal to Noise Ratio by: $\text{SNR} = 20\log_{10}(\rho_0/2\sigma)$ (dB). The measurements' acquisition time is assumed to be greater than the coherence time of the target, the phase φ_k is random here. It is then better to work on real samples $z_k^{lm} = |r_k^{lm}|^2$. Let's note that the samples z_k^{lm}/σ^2 are distributed according to a χ_2^2 either centered if $s_k = 0$, or noncentral if $s_k = 1$ with the noncentrality parameter $\lambda^{lm} = |\rho_k \, h^{lm}(\mathbf{x}_k)|^2/\sigma^2$, the probability density z_k^{lm} can be written as :

$$p(z_k^{lm}|\mathbf{X}_k) = \frac{1}{2\sigma^2} e^{-\frac{z_k^{lm}}{2\sigma^2}} \left[e^{-\frac{\lambda^{lm}}{2}} I_0\left(\sqrt{\frac{\lambda^{lm} z_k^{lm}}{\sigma^2}}\right)\right]^{s_k}, \quad (6)$$

where $I_0(.)$ is the modified Bessel function of first kind. The samples z_k^{lm} being assumed to be independent, the probability distribution of all observations, denoted by $\mathbf{z}_k = \{z_k^{lm} : l = 1, \ldots, N_d, m = 1, \ldots, N_\theta\}$, is then written as:

$$p(\mathbf{z}_k|\mathbf{X}_k) = \prod_{l=1}^{N_d} \prod_{m=1}^{N_\theta} p(z_k^{lm}|\mathbf{X}_k). \quad (7)$$

The radar problem's modeling discussed above reveals high non-linearities and possibly non-Gaussianity, the classical tracking strategy must be redesigned, the solutions of the Kalman family cannot here perform efficiently. In this paper, a particle filter will be used to perform the TBD and solve our problem.

3 Particle filter optimized in TBD

The particle filter approximates the probability density function (pdf) of the state $p(\mathbf{X}_k|\mathbf{z}_{1:k})$ using a stochastic particles cloud $\{\mathbf{X}_k^i, w_k^i\}_{i=1,..,N}$ where $\mathbf{X}_k^i = \left[\mathbf{x}_k^i, \rho_k^i, s_k^i\right]^T$ and w_k^i are respectively the state and the importance weights assigned to each particle i. These particles are propagated over time using a proposal density $q(\mathbf{X}_k|\mathbf{X}_{k-1}, \mathbf{z}_k)$. The special feature of the particle filter in TBD [5, 8] comes from the existence of particles in different modes s_k, which can evolve over time. This kind of filter appears to be very sensitive to the choice of the proposal density, and particularly the law used for the particles "birth" (case $s_{k-1}^i = 0$ and $s_k^i = 1$).

3.1 Choice of the proposal density

We propose in this paper a particle filter based on a proposal density factorizes as :

$$q(\mathbf{X}_k|\mathbf{X}_{k-1}, \mathbf{z}_k) = q(s_k|s_{k-1})\; q_{\mathbf{x}_k}\; q_{\rho_k} \tag{8}$$

where

$$q_{\mathbf{x}_k} = q(\mathbf{x}_k|\mathbf{x}_{k-1}, s_k, s_{k-1}, \mathbf{z}_k),$$
$$q_{\rho_k} = q(\rho_k|\rho_{k-1}, \mathbf{x}_k, s_k, s_{k-1}, \mathbf{z}_k).$$

$q(s_k|s_{k-1})$ is the *prior* $p(s_k|s_{k-1})$. For the set of continuing particles, $q_{\mathbf{x}_k}$ and q_{ρ_k} are classically provided by applying the dynamic model (2). However the knowledge of the observations $\mathbf{z}_k$ and the past state $\mathbf{X}_{k-1}$ is used to influence the initial state of the particles $[\mathbf{x}_k, \rho_k]$; this case is the most sensitive in terms of performance. This strategy differs from the usual choice done in the literature, which uses as proposal density the prior density of the particle states. The data $\mathbf{z}_k$ could conceivably be used to concentrate immediately the birth particles in a region of interest of the state space, and thus reduce volume of the state space to be sampled. The proposal density considered for the birth particles state $\mathbf{x}_k$ is defined as :

$$q(\mathbf{x}_k|\mathbf{x}_{k-1}, s_k = 1, s_{k-1} = 0, \mathbf{z}_k) = \mathcal{U}_\mathcal{D}, \tag{9}$$

where $\mathcal{U}_\mathcal{D}$ is a uniform distribution within $\mathcal{D} = \{(l,m)|z_k^{lm} > \gamma\}$ all the resolution cells, in which the signal measured is higher than a threshold of detection γ. This threshold is given by $\gamma = -2\sigma^2 \log P_{fa}$, where P_{fa} represents the probability of false alarm per resolution cell.

The choice of P_{fa} requires a compromise between volume to be sampled and the probability to initialize at least one particle in the cell containing the target (which also increase with P_{fa}). In practice, choosing a probability of detection per cell of $P_D \approx 0.9$ for SNR higher than 7 dB results in $P_{fa} \approx 0.1$ and reduces the state space of about a factor ten.

We propose here to sample the amplitude according the following distribution :

$$q(\rho_k|\mathbf{x}_k, s_k = 1, s_{k-1} = 0, \mathbf{z}_k) = \mathcal{N}(\hat{\rho}(\mathbf{x}_k), \sigma_\rho^2), \tag{10}$$

i.e. a normal distribution with mean $\hat{\rho}(\mathbf{x}_k)$ provided by an estimate of the module of the complex amplitude ρ_k associated to a target's state $\mathbf{x}_k$, and variance σ_ρ^2 such as $\sigma_\rho \ll \rho_{max} - \rho_{min}$. In practice, computing the particles' weight appears to be robust to a poor estimate of ρ_k. For each birth particle $\mathbf{X}_k^i$, we use the following estimate :

$$\hat{\rho}(\mathbf{x}_k^i) = \sqrt{\max\left(\frac{z_k^{l_c m_c} - 2\sigma^2}{\left(h^{l_c m_c}(\mathbf{x}_k^i)\right)^2}, \rho_{min}^2\right)} \tag{11}$$

where $(l_c, m_c) = \underset{(l,m)}{\arg\max}\, h^{lm}(\mathbf{x}_k^i)$, is the resolution cell in which is the particle. One can choose another proposal density, for instance a uniform distribution on an interval centered at $\hat{\rho}(\mathbf{x}_k)$, in order to reduce the size of space to sample.

After division by the constant $p(\mathbf{z}_k|s_k = 0)$, which leads to importance weights equal to 1 if $s_k^i = 0$, weights can be expressed by the equation 13.

Note that the choice of the proposal density can strongly influence the birth particles weight. Indeed, choose σ_ρ^2 or P_{fa} very low leads to a significant reduction of the state space, but also penalizes heavily the birth particles weight via the compensation ratio required by the weight calculations. A trade-off between reducing the state space to be sampled and improving the performance must be considered.

3.2 Minimum number of particles required

We propose in this section a method to determine a minimum number of particles,which ensures good detection performance. This method is based on the characteristics of the proposal density described in previous section. Let's denote N_{fa} the number of cells such as $z_k^{lm} > \gamma$, where γ guarantees a probability of detection P_D per resolution cell for a fixed SNR. N_{fa} follows a binomial distribution $\mathfrak{B}(N, P_{fa})$, where P_{fa} is the probability of false alarm associated with P_D. We would like to give birth to at least one particle per cell N_{fa} with some probability P, to ensure the birth of one particle in the cell in which is the target with the same probability P. Let's denote α_0 the particles' proportion with state $s_k = 0$ when there is no target, steady-state filter case and N_p the particles number, we give birth to an average of $N_n = \alpha_0 P_b N_p$ particles at each step. We then look for N_n such as $P(N_{fa} \leq N_n) \geq P$. It follows :

$$N_{min} \approx \frac{\underset{N_n}{\arg\min}\, \{P(N_{fa} \leq N_n) \geq P\}}{P_n \alpha_0} = \frac{F_{\mathfrak{B}}^{-1}(P|N, P_{fa})}{P_n \alpha_0}, \tag{12}$$

where $F_{\mathfrak{B}}^{-1}(.|N, P_{fa})$ is the inverse cumulative distribution function of N_{fa}. Trends in the required particles number as a function of SNR is shown in figure 1 for various P_D.

$$w_k^i \propto \begin{cases} \frac{p(\mathbf{z}_k|\mathbf{x}_k^i, \rho_k^i, s_k^i=1)}{p(\mathbf{z}_k|s_k=0)} & \text{si } s_k^i = 1 \text{ et } s_{k-1}^i = 1 \\ \frac{p(\mathbf{z}_k|\mathbf{x}_k^i, \rho_k^i, s_k^i=1)}{p(\mathbf{z}_k|s_k=0)} \frac{p(\mathbf{x}_k^i|s_k^i=1, s_{k-1}^i=0)}{q(\mathbf{x}_k^i|s_k^i=1, s_{k-1}^i=0, \mathbf{z}_k)} \frac{p(\rho_k^i|s_k^i=1, s_{k-1}^i=0)}{q(\rho_k^i|\mathbf{x}_k^i, s_k^i=1, s_{k-1}^i=0, \mathbf{z}_k)} & \text{si } s_k^i = 1 \text{ et } s_{k-1}^i = 0 \end{cases} \tag{13}$$

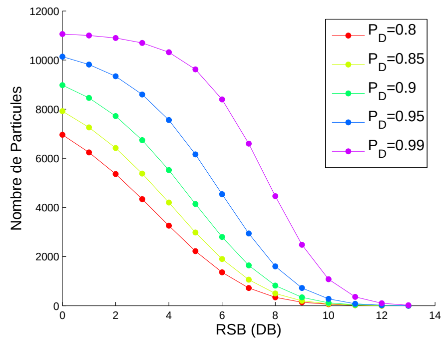

FIG. 1: Number of particles required as a function of SNR for different P_D with $N = 560$, $\alpha_0 = 0.5$ and $P_n = 0.1$.

4 Results and Conclusion

In this section we give a demonstration of our particle filter that is capable of detecting and tracking a point target flying at a constant radial velocity with a constant amplitude corresponding to an SNR of 7 dB. In the scenario, initially there is no target present, the target appears after $t = 10$s and disappears at time step $t = 75$s. We consider the following parameters for the dynamic model : $P_n = P_m = 0.1$, $T = 1$ s, $v_{max} = -v_{min} = 0.1\,\text{km.s}^{-1}$, $\rho_{min} = 2\,\text{dB}$, $\rho_{max} = 20\,\text{dB}$, $q_s = q_i = 10^{-3}$. The measurements are simulated with the following parameters: $d_{min} = 100\,\text{km}$, $\theta_{min} = -10\,\text{degree}$, $\Delta_d = 0.5\,\text{km}$, $\Delta_\theta = 1.45\,\text{degree}$, $N_d = 40$, $N_\theta = 14$, $\sigma^2 = 0.5$, $B = 150\,\text{kHz}$, $T_e = 6.67 \times 10^{-5}\,\text{s}$, $N_a = 70$, $\lambda_P = 3\,\text{cm}$, $d_a = \lambda_P/2$, $c = 3 \times 10^8\,\text{m.s}^{-1}$.

The figure 2-(a) presents the estimate of s_k (which corresponds to the estimated probability of presence) gauged on an average over $N_{MC} = 1000$ Monte-Carlo runs. The results obtained with our filter are compared with those of a conventional filter using either the prior density of the particle's states, or a uniformly distribution within the highest power bins for the birth particles [8]. Let consider the following parameters : $P_{fa} = 0.1$, $\sigma_\rho = 0.1$ et $N_p = 1000$. Our algorithm gives improved detection performance, both in presence of a target and without any target. The observed oscillations are due to losses to $3dB$ at the edge of resolution cell. It was also resulted in an improved accuracy of the estimate of the target's state.

The figure 2-(b) shows the average probability of existence calculated by our filter with different particles numbers. The performances obtained with the theoretical minimum number of particles described above, corresponding to $P_D = 0.90$, $P = 0.99$ and SNR of 7 dB, i.e. 1640 particles, are actually almost optimal.

This paper has highlighted the importance of choosing an adapted and informative proposal density in term of performance of a particle filter designed for recursive track-before-detect. Our filter, based on relevant choices about proposal densities, leads to a simple, fast and efficient algorithm. The next step in this work will consist in extending this approach to be able of detecting and tracking multiple targets.

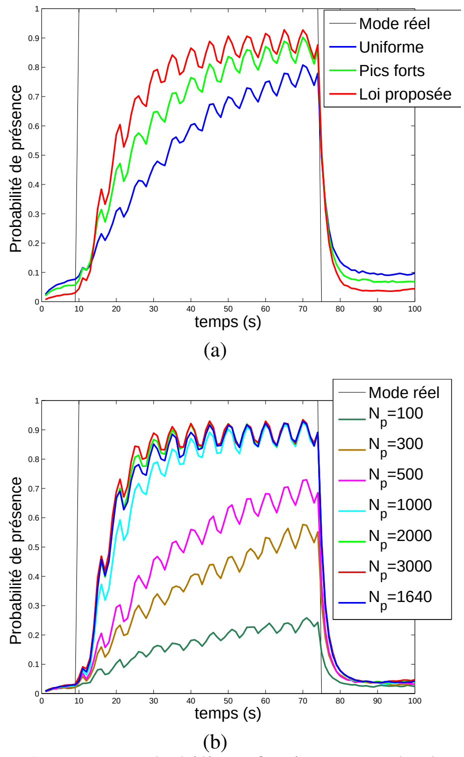

FIG. 2: (a) : Average probability of existence calculated for the three proposal density variations. (b) : Probability of existence as a function of the particles'number for a fixed SNR of 7 dB.

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