Multitarget Particle filter addressing Ambiguous Radar data in TBD

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***Abstract*— In this paper, we have addressed the problem of multiple target tracking in Track-Before-Detect (TBD) context using ambiguous Radar data. TBD is a method which uses raw measurement data, i.e. reflected target power, to track targets. Tracking can be defined as the estimation of the state of a moving object based on measurements. These measurements are in this case assumed to be the radar echoes ambiguous in range and doppler. The estimated states are produced by means of a tracking filter. The filtering problem has been solved by using a Particle Filter (PF). Particle filtering is a signal processing methodology, which can effectively deal with nonlinear and non-Gaussian signals by a sample-based approximation of the state probability density function (pdf).**

A standard multitarget SIR Particle Filter is extended so that it can handle range/doppler ambiguities and eclipsing effects. Such extension is required for its use in practice and to enhance tracking accuracy. The proposed particle filter succeeds in resolving range and doppler ambiguities, detecting and tracking multiple targets in a TBD context.

I. INTRODUCTION

Multitarget tracking is a well-known problem which consists of sequentially estimating the states of several targets from noisy data. It is encountered in many applications, e.g. aircraft tracking from radar measurements. Classical radar tracking methods take thresholded measurements, so called plots, as an input. When the energy reflected by a target is too low, no plot can be constructed and this target will therefore be declared lost. To overcome this problem a method, called Track before Detect (TBD), has been developed, see [1], [2]. The TBD approach proposes to skip the pre-detection step used to isolate detection plots and thus bases the tracking on the raw measurements instead of plots. In the TBD approach, the detection is performed at the end of the processing chain, i.e. when all information has been gathered and integrated over time. This way information on the estimated state of a possible target is used in the detection problem.

A. Integrated Detection and Tracking Processing

The first problem that arises with the TBD application is the tracking problem. The tracking is based on recursive estimation of the target state vector from a series of noisy observations. This vector is usually described by a state model that governs the dynamics of the target, while the observation is described by a measurement equation. In a Bayesian framework, the estimates are derived from the probability density of the state vector, inferred from these two models, conditioned on the measured data.

However, e.g. in TBD, the state and observation models have strong nonlinearities. The classical tracking strategy must be redesigned. Solutions using a Particle Filter (PF) have been proposed in the past fifteen years [3], [4]. Particle methods are a set of Sequential Monte Carlo (SMC) simulation-based methods numerically approximating the posterior filtered density. The PF has demonstrated its ability to perform successful nonlinear and non-Gaussian tracking and to enable more accurate modeling of the dynamics of physical systems and measurement devices. An excellent tutorial has been published on the subject [5].

Apart from the problem of filtering a signal the second problem which arises is the detection problem. In TBD, the detection consists of the processing of radar reflections (reflected target energy plus noise) and deciding whether these radar reflections are target originated or not.

B. Ambiguity resolution

Pulse-Doppler signal processing also includes ambiguity resolution to identify true range and velocity. Radar systems employ low, medium, and high PRF schemes. The fold range (in km) that can be observed using a specific PRF (in kHz) is given by $150/PRF$. The fold velocity (in m/s) is related to the PRF (in kHz) as $150PRF/f_c$, where f_c is the carrier frequency (in GHz).

In general, we would like to be able to choose the PRF to provide the desired unambiguous range and Doppler bandwidth simultaneously. However, unambiguous range increases with decreasing PRF, while unambiguous Doppler increases with increasing PRF.

Ambiguity resolution finds true range and true speed by using sequential measurement of the ambiguous range and Doppler at each burst. The comparison of these measurements will then serve to eliminate ambiguities. The most common method for resolving range and Doppler ambiguities involves using multiple PRFs. This has the effect of changing the apparent target range estimated by each pulse, or pulse burst, and allows for some ambiguity resolution in either Doppler or range, depending on the particular application and prevents blind spots in the range and in the radial velocity coverage. The classical solution to the PRF selection problem is to perform

PRF staggering (or PRF jittering) [6], meaning the repetitive (or random) use of at least three different PRFs.

In this paper we focus on modeling issues arising from range/doppler ambiguities and eclipsing effects in practice with radar systems. Thus, we propose a PF allowing for tracking, detection and ambiguity problems to be solved over time in a TBD context. This work exploits integrated Monte Carlo processing to enhance tracking accuracy by including ambiguities and eclipsing effects to identify true range and velocity.

The paper is organized as follows: section II briefly reviews some radar fundamentals in analyzing and understanding the ambiguity and eclipsing issues associated with radar systems analysis and design principles; in section III the system dynamics and measurement model, that includes ambiguities and eclipsing effects, are introduced for the specific TBD surveillance application. Section IV introduces a PF designed for our TBD application. Section V collects our simulations results. We demonstrate through simulations the practical abilities of the proposed algorithm to improve performance under conditions where ambiguity and eclipsing effects are significant. Finally, we report our conclusions and direction for future research in section VI.

II. Radar Fundamentals : Ambiguities and Eclipsing Effects

The target's range is measured by transmitting a pulse of radio-frequency (RF) energy with a pulse width τ_{pulse}. This pulse is reflected by the target back to the radar after a time delay. By measuring the round-trip travel time of this pulse, we can determine the distance between the radar and the target. However, the raw return signal from a reflection will appear to be coming from a distance less than the true range (r_{true}) of the reflection when the range of the reflection exceeds the wavelength of the pulse repetition frequency (PRF). This causes reflected signals to be folded, so that the apparent range (r_{app}) is a modulo function of the true range. Once a pulse is transmitted, to avoid range ambiguity the radar should wait till the echoes of targets at maximum range are back before transmitting the next pulse. It follows that the range measurements correspond to:

$$r_{app} \equiv r_{true} \left(\bmod \frac{c}{2 \text{ PRF}} \right). \tag{1}$$

Pulse eclipsing occurs due to the receiver being switched off while the radar is transmitting another pulse. Therefore the entire waveform will not be received for targets with a time delay approximatively equal to the pulse repetition interval(PRI), that are very close to the transmitting system or near the end of the range-unambiguous extent. Each individual PRF has blind ranges areas, where the transmitter pulse occurs at the same time as the target reflection signal arrives back at the radar, of width:

$$r_{ecl} = \tau_{pulse} \cdot c. \tag{2}$$

Eclipsing is a particular problem in systems with a high duty ratio, where the length of the pulse is large compared to the pulse repetition interval, because a large proportion of the range profile will be eclipsed.

Doppler radars have the ability to extract target radial speed to distinguish moving from stationary targets, such as clutter, by measuring the doppler shift that a target imposes on the reflected pulse. In each pulse the frequency shift cannot be used directly, but one can measure the phase variation from pulse to pulse given by:

$$\Delta_{\varphi} \equiv \frac{-2\pi \ v_{true}}{\text{PRF } \lambda} \pmod{2\pi}, \tag{3}$$

where λ is the wavelength of the transmitted energy. This introduces a modulo operation onto the apparent frequency of the reflected signal and causes reflected signals to be folded for high speed targets where radial velocity produces a frequency shift above the PRF, so that the apparent radial velocity (v_{app}) is a modulo function of true radial velocity (v_{true}):

$$v_{app} \equiv v_{true} \left(\bmod \frac{\text{PRF } \lambda}{2} \right). \tag{4}$$

A blind velocity occurs when the Doppler frequency falls close to the PRF. This folds the return signal into the same filter as stationary clutter reflections. Each individual PRF has blind velocities where the velocity of the target will appear stationary:

$$d_{ecl} = n \ \frac{\lambda \text{ PRF}}{2}, \text{ where } n \in \mathbb{N}^*. \tag{5}$$

III. System Setup and TBD Problem Formulation

In this section, we will describe the system dynamic model and the measurement model.

Note that we will assume throughout this paper that the number of targets M is unknown and variable over time. Let us now denote by $\mathbf{x}_{k,m}$ the vector describing the state of the m^{th} target ($m \in \{1, M\}$) at time step k written as:

$$\mathbf{x}_{k,m} = [\mathbf{s}_{k,m}, \rho_{k,m}, m_{k,m}]^T \tag{6}$$

where, $\mathbf{s}_{k,m}$ represents the position and velocity of the m^{th} target in Cartesian coordinates and $\rho_{k,m}$ is the unknown modulus of the target complex amplitude and $m_{k,m}$ is a binary variable representing the presence ($m_{k,m} = 1$) or the absence ($m_{k,m} = 0$) of the target t. The state vectors $(\mathbf{x}_{k,m})_{m\in\{1,M\}}$ can be concatenated into the multitarget state vector $\mathbf{x}_k$.

A. Dynamic model

To model the dynamics of the targets we adopt a nearly constant velocity model to describe object position and velocity in a Cartesian frame, see e.g. [3], and a random walk model for object amplitude $\rho_{k,m}$. The model uncertainty is handled by the process noise $\mathbf{v}_{k,m}$, which is assumed to be standard white Gaussian noise with covariance G. Under these assumptions, the corresponding state-space model, for $\mathbf{x}_{k,m}$, the state vector associated to the m^{th} target, is given by:

$$\mathbf{x}_{k+1,m} = F\mathbf{x}_{k,m} + \mathbf{v}_{k,m}, \tag{7}$$

where F represents a transition matrix with a constant sampling time T such as:

$$\begin{aligned} F &= \operatorname{diag}(F_1, F_1, 1), \\ \text{where, } F_1 &= \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}. \end{aligned} \tag{8}$$

and G, the covariance process noise, is given by:

$$\begin{aligned} G &= \operatorname{diag}(a_x G_1, a_y G_1, a_\rho T), \\ \text{where, } G_1 &= \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^2}{2} & T \end{bmatrix}, \end{aligned} \tag{9}$$

where $a_x = a_y$ and a_ρ denote the level of process noise in object motion and amplitude, respectively. This model correctly approximates small accelerations in the object motion and fluctuations in the object amplitude.

In the hybrid state estimation problem the birth / death (appearance / disappearance) process of a target is modeled according to a jump Markov process, whose transition probabilities P_b and P_d.

$$\begin{aligned} P_b &= P(m_{k,m} = 1 \mid m_{k-1,m} = 0) \\ P_d &= P(m_{k,m} = 0 \mid m_{k-1,m} = 1) \end{aligned} \tag{10}$$

denote the probabilities of target birth and death, see [8]. When $m_{k-1,m} = 0$, the position, velocity and $\rho_{k,m}$ are respectively uniformly distributed over the observation window of the radar, the block $[s_{min}, s_{max}]^2$ and the interval $[\rho_{min}, \rho_{max}]$.

B. Measurement model

Typical radar measurements include range, doppler speed, bearing and elevation angles. We assume to have a system that measures the range (r), the doppler speed (d) and the bearing angle (b) of a target. At discrete instants k, the radar system positioned at the Cartesian origin collects a noisy signal. Each measurement $\mathbf{z}_k$ consists of $N_r \times N_d \times N_b$ reflected power measurements $\mathbf{z}_k^{ijl}$, where N_r, N_d and N_b are the number of range, doppler and bearing cells.

Each resolution cell (i, j, l) is centered in :
$\left(r_{min} + \left(i - \frac{1}{2}\right)\Delta_r, d_{min} + \left(j - \frac{1}{2}\right)\Delta_d, b_{min} + \left(l - \frac{1}{2}\right)\Delta_b\right)$
where Δ_r, Δ_d and Δ_b represent respectively range, doppler and bearing resolutions associated to the radar; r_{min}, d_{min} and b_{min} are respectively the minimum range, minimum doppler and minimum bearing reported by the radar. The range and doppler resolutions are given by the following equations:

$$\Delta_r = PCR \, \frac{\tau_{pulse} \cdot c}{2} \tag{11}$$

$$\Delta_d = \frac{\lambda \cdot PRF}{2 \cdot n_{pulses}} \tag{12}$$

where PCR and n_{pulses} correspond respectively to the pulse compression factor and the number of transmitted pulses.

The power measurement per range-doppler-bearing cell is defined by :

$$\mathbf{z}_k^{ijl} = |\mathbf{z}_{\rho,k}^{ijl}|^2, k \in \mathbb{N}. \tag{13}$$

where $\mathbf{z}_{\rho,k}^{ijl}$ is the complex envelope data of the target described by the following nonlinear equation, see [3]:

$$\mathbf{z}_{\rho,k}^{ijl} = \sum_{m=1}^{M} m_{k,m} \, \rho_{k,m} \, e^{i\varphi_k} h^{ijl}(\mathbf{s}_{k,m}) + \mathbf{w}_k^{ijl}, \quad \varphi_k \in (0, 2\pi) \tag{14}$$

where $h^{ijl}(\mathbf{s}_{k,m})$ is the reflection form of the m^{th} target, that for every range-doppler-bearing cell is defined by:

$$h^{ijl}(\mathbf{s}_{k,m}) := e^{-\frac{(r_i - r_k^m)^2}{2R} - \frac{(d_j - d_k^m)^2}{2D} - \frac{(b_l - b_k^m)^2}{2B}}, \tag{15}$$
$$i = 1, \ldots, N_r, \; j = 1, \ldots, N_d, \; l = 1, \ldots, N_b \text{ and } k \in \mathbb{N}$$

where the relationship between the measurement space and the target space can be established as: Apparent target range :

$$r = \sqrt{x^2 + y^2} \left(\bmod \frac{c}{2 \text{ PRF}}\right); \tag{16}$$

Apparent target doppler :

$$d = \frac{(x \; v_x + y \; v_y)}{\sqrt{x^2 + y^2}} \left(\bmod \frac{\text{PRF } \lambda}{2}\right); \tag{17}$$

Target azimuth :

$$b = \arctan\left(\frac{y}{x}\right). \tag{18}$$

The reflection form describes the height of the target signal amplitude in the cells surrounding the target. R, D and B are related to the size of a range, a doppler and a bearing cell. $\mathbf{w}_k^{ijl}$ are independent samples of a complex white Gaussian noise with variance $2\sigma_w^2$, (*i.e.* the real and imaginary components are assumed to be independent, zero-mean white Gaussian with the same variance σ_w^2).

Furthermore, the eclipsing effect is considered, so the reflection form of a target whose apparent range fall in a blind zone will be assumed null.

C. TBD Problem Formulation

Consider the system represented by the equations (7), (13) and (14). Assume that the set of measurements collected up to the current time is denoted by $\mathbf{Z}_k = \{\mathbf{z}_1, \ldots, \mathbf{z}_k\}$. The filtering problem can be formulated as finding the a posteriori distribution of the joint state $\mathbf{x}_k$ for all possible numbers of targets conditioned on all past measurements $\mathbf{Z}_k$.

The radar problem's modeling discussed above reveals high nonlinearities. The classical tracking strategy must be re-designed, because the solutions of the Kalman family cannot perform efficiently here. In this paper, a particle filter will be used to perform the TBD and solve our problem.

IV. Bayesian inference and PF solution in TBD

A. Bayesian Filtering Problem

Given a realization of $\mathbf{Z}_k$, Bayesian filtering [9] consists of finding an approximation of the posterior pdf $p(\mathbf{x}_k|\mathbf{Z}_k)$. In particular, the *marginal filtering distribution* $p(\mathbf{x}_k|\mathbf{Z}_k)$ is obtained at time k by means of a two-step recursion:

(1) the Prediction Step, which is solved using the *Chapman-Kolmogorov equation*, i.e.,

$$p(\mathbf{x}_k|\mathbf{Z}_{k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1}) \; p(\mathbf{x}_{k-1}|\mathbf{Z}_{k-1}) d\mathbf{x}_{k-1} \tag{19}$$

where, $p(\mathbf{x}_k|\mathbf{Z}_{k-1})$ is the predictive density at time k;
(2) the Update Step, which is solved using *Bayes theorem*, i.e.,

$$p(\mathbf{x}_k|\mathbf{Z}_k) = \frac{p(\mathbf{z}_k|\mathbf{x}_k)\ p(\mathbf{x}_k|\mathbf{Z}_{k-1})}{p(\mathbf{z}_k|\mathbf{Z}_{k-1})} \tag{20}$$

where $p(\mathbf{z}_k|\mathbf{Z}_{k-1})$ is the Bayes normalization constant (*evidence*). Thus, the filtering pdf is completely specified given some prior $p(\mathbf{x}_0)$, a transition kernel $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ and a likelihood $p(\mathbf{z}_k|\mathbf{x}_k)$.

B. Particle filter solution in TBD

The detailed PF algorithm, based on [3], is described below and summarized in Algorithm 1. The particle filter approximates the a posteriori pdf of the state $p(\mathbf{x}_k|\mathbf{Z}_k)$ by approximately solving the Bayes prediction and update equations recursively using a stochastic particles cloud. At time step $k-1$, the posterior pdf $p(\mathbf{x}_{k-1}|\mathbf{Z}_{k-1})$ is approximated by a set of N_p particles $\{\mathbf{x}_{k-1}^{(i)}\}_{i=1}^{N_p}$ with associated importance weights $\{w_{k-1}^{(i)}\}_{i=1}^{N_p}$. Each particle contains the joint state of at most M_{max} partitions, each one corresponding to a different target. We refer to $\mathbf{x}_{k-1,m}^{(i)}$ as the states of the m^{th} partition of the particle i at time step $k-1$ defined only for $e_{k-1,m}^{(i)}$. The particle state vector associated to M targets is given by :

$$\mathbf{x}_{k-1}^{(i)} = \left[\mathbf{x}_{k-1,1}^{(i)}, \ldots, \mathbf{x}_{k-1,M}^{(i)}\right].$$

Each particle $\mathbf{x}_{k-1}^{(i)}$ is passed through the system dynamics eq.(7) to obtain $\{\hat{\mathbf{x}}_k^{(i)}\}_{i=1}^{N_p}$, the predicted particles at time step k. Once the observation data $\mathbf{z}_k$ is received, the importance weight of each predicted particle can be evaluated. The special feature of the PF in TBD, see [1], [8], comes from the existence of particles in different modes $e_{k-1,m}^{(i)}$, which represents the discrete possibly time variable mode of the partition m of the particle i.
Then, after normalization of the weights, the a posteriori density $p(\mathbf{x}_k|\mathbf{Z}_k)$ at time step k can be approximated by the empirical distribution as :

$$\hat{p}(\mathbf{x}_k|\mathbf{Z}_k) := \sum_{i=1}^{N_p} w_k^{(i)}\ \delta_{\hat{\mathbf{x}}_k^{(i)}}(\mathbf{x}_k),$$

where $\delta_{\hat{\mathbf{x}}_k^{(i)}}(.)$ denotes the delta-Dirac mass located at $\hat{\mathbf{x}}_k^{(i)}$. In fact, the standard algorithm guarantees convergence in an almost sure sense, of the empirical distribution to the true, but unknown, a posteriori distribution of interest $p(\mathbf{x}_k|\mathbf{Z}_k)$, if the number of particles, N_p, tends to infinity. See [10] for a proof and [11] for more detail and an overview of convergence results.
Different and more efficient algorithms for multi target particle filters exist, see e.g. [12].

C. Likelihood description

The likelihood expression in our particle filtering TBD application on range bearing doppler data, is based on the SW I/II fluctuation model. The power of a target echo in one range-Doppler-bearing cell, $\mathbf{z}_k^{ijl}$, conditioned on the state, $\mathbf{x}_k$, is assumed to follow the exponential distribution :

$$p(\mathbf{z}_k^{ijl}|\mathbf{x}_k) = \frac{1}{\mu_0^{ijl}} e^{-\frac{1}{\mu_0^{ijl}}\ \mathbf{z}_k^{ijl}} \tag{21}$$

where,

$$\begin{aligned} \mu_0^{ijl} &= \mathbf{E}\left[\mathbf{z}_k^{ijl}\right] \\ &= \sum_{m=1}^{M} m_{k,m}\rho_{k,m}{}^2 \left(h^{ijl}(\mathbf{s}_{k,m})\right)^2 + 2\sigma_w^2. \end{aligned} \tag{22}$$

Additionally, the target echo is assumed to be independent from cell to cell. The latter is a robust assumption that is required at least in burst-wise RF-agility systems, where the target echo indeed fluctuates from burst to burst. These assumptions lead to a simple likelihood expression for the measured power conditioned on the state, i.e. it results in simple expressions for calculating the likelihood of each particle. The conditional likelihood of the received data according to the number of targets present is written as :

$$p(\mathbf{z}_k|\mathbf{x}_k) = \prod_{i,j,l} p(\mathbf{z}_k^{ijl}|\mathbf{x}_k). \tag{23}$$

Algorithm 1: SIR Multi target outline

input : $\{\mathbf{x}_{k-1}^{(i)}, w_{k-1}^{(i)}\}_{i=1}^{N_p}$ and a new measurement, $\mathbf{z}_k$.
output: $\{\mathbf{x}_k^{(i)}, w_k^{(i)}\}_{i=1}^{N_p}$.

Initialization:
Sample initial particles $\{\mathbf{x}_0^{(i)}\}_{i=1}^{N_p}$ from $p(\mathbf{x}_0)$;
Set the weights $w_0^{(i)}$ to $\frac{1}{N_p}$.

1 - *Prediction:*
while $i \leftarrow 1$ **to** N_p **do**
  Generate a new particle : $\hat{\mathbf{x}}_k^{(i)} = F\mathbf{x}_{k-1}^{(i)} + \mathbf{v}_{k-1}^{(i)}$.
end

2 - *Update:*
while $i \leftarrow 1$ **to** N_p **do**
  Compute weights : $\tilde{w}_k^{(i)} = p(\mathbf{z}_k|\hat{\mathbf{x}}_k^{(i)})$;
end
Normalize the weights : $w_k^{(i)} = \frac{\tilde{w}_k^{(i)}}{\sum_{j=1}^{N_p} \tilde{w}_k^{(j)}}$.

3 - *Resample:*
Effective Sample Size: $N_{eff} = \frac{1}{\sum_{i=1}^{N_p} (w_k^{(i)})^2}$;
if $N_{eff} \leq \beta N_p$ **then**
  Generate a new set of particles $\{\mathbf{x}_k^{(j)}\}_{j=1}^{N_p}$,
  so that for any j, $P(\mathbf{x}_k^{(j)} = \hat{\mathbf{x}}_k^{(i)}) = w_k^{(i)}$;
  Set the weights $w_k^{(i)}$ to $\frac{1}{N_p}$.
end

V. SIMULATIONS

Simulations are performed with a simple scenario to illustrate the solving ambiguities problem for detection and tracking purposes.

A. Experiment setup

We consider a scenario where two targets have to be tracked by a radar that can use several PRFs. In the simulated scenario, the radar is assumed to be at the origin of the axes and the target are assumed to move radially to the sensor with a constant velocity and without any maneuvers. The received power of the targets are assumed unknown and can fluctuate from scan to scan. Therefore, we use a second order tracking model and consider a SW I measurement model.
We assume that the targets will be tracked in a Cartesian frame. Initially there is no target present, the first target appears after 5 seconds at a position of $[x_0^1, y_0^1] = [70, 40]$ km from the sensor and flies at a constant velocity of $[v_{x0}^1, v_{y0}^1] = [-130, -75]$ m.s^{-1} directly to the sensor. Its SNR is assumed to be 11 dB. At $k = 8$ a second target appears at a position of $[x_0^2, y_0^2] = [86, 50]$ km from the radar and moves with constant velocity of $[v_{x0}^1, v_{y0}^1] = [-216, -125]$ m.s^{-1} towards the radar. Its SNR is assumed to be 7 dB.

We consider two multiple PRF schemes :

1) Radar can use a single PRF in one scan and resolve ambiguity in the next step by changing the PRF;
2) Radar can use an increasing (change) of the PRF within each dwell interval(interval pulse period).

The selection of the PRF directly affects the maximum fold range and doppler respectively r_{fold} and d_{fold}; the range and doppler resolutions associated to the radar respectively Δ_r and Δ_d; the blind range areas width r_{ecl} and the blind velocities d_{ecl}. In this scenario, the radar can switch between three different PRFs. The corresponding settings associated with each of the chosen PRFs are listed in Table I :

TABLE I
THE CHOSEN PRFS AND THEIR CORRESPONDING SETTINGS

PRF[kHz]	3	4	5
PCR [%]	2.7	3.6	4.5
λ [m]	0.030	-	-
τ_{pulse} [μsec]	23	19	15
n_{pulses}	6	8	10
r_{fold} [km]	49.96	37.47	29.98
Δ_r [m]	93.08	102.53	101.18
r_{ecl} [km]	6.89	5.69	4.49
d_{fold} [m.s^{-1}]	45	60	75
Δ_d [m.s^{-1}]	7.5	-	-

Initially, 3000 particles are uniformly distributed in the state space, in an area between $[0, 105]$ km, $[-240, 0]$ m.s^{-1} in x-direction and $[0, 60]$ km, $[-150, 0]$ m.s^{-1} in the y-direction with amplitude sample according a normal distribution with mean $\hat{\rho}(\mathbf{s}_{k,m})$ and variance σ_ρ^2 such as $\sigma_\rho \ll \rho_{max} - \rho_{min}$, where $[\rho_{min}, \rho_{max}] \in [1.5849, 6.3096]$, with SNR $\in [4, 16]$ dB and a level of noise $\sigma_n = \frac{1}{2}$.
Furthermore, the modes in each setup are equally divided among the number of particles. We consider a jump Markov process, whose transition probabilities $P_b = P_d = 0.2$. Birth particles are uniformly distributed within the highest power bins, whereas the state prediction of remaining particles is done according to the dynamic model eq.(7). The update time, T, is set to 1 sec. The dynamics of both targets are captured by a constant velocity model with $a_x = a_y = 30$ m.s^{-1} and $a_\rho = 10$ mW. During the simulation the particles are constrained to be in the measurement space, i.e. when the predicted state of a particle is outside this space the weight of the particle is set to zero.
Besides a state space we have a measurement space, i.e. range, doppler and bearing measurements. We consider range cells in the interval $[0, 120]$ km and doppler speed cells in the interval $[-300, 0]$ m.s^{-1}. Because we assume that the target moves radially to the sensor the bearing angle is small enough that it can be represented with a single bearing cell $b^m \in \frac{\pi}{6} \pm \Delta_b$ Rad with $\Delta_b = 1.2$ mRad. The unambiguous measurement space is therefore divided into $N_r \times N_d \times N_b$ cells, where $N_r = [537\ 366\ 297]$, $N_d = [7\ 9\ 11]$ and $N_b = 1$.
Figure 1 shows the measurement data in the range and doppler cells of this scenario at different points in time $k = 7$ and 20. Notice the reflection form of the target with a SNR of 11 dB, this moves from one end of the range cell to the other but remains in the same doppler cells. This indicates that the target is moving to sensor with a constant doppler speed. Here the target with a SNR of 7 dB is completely lost in the noise.

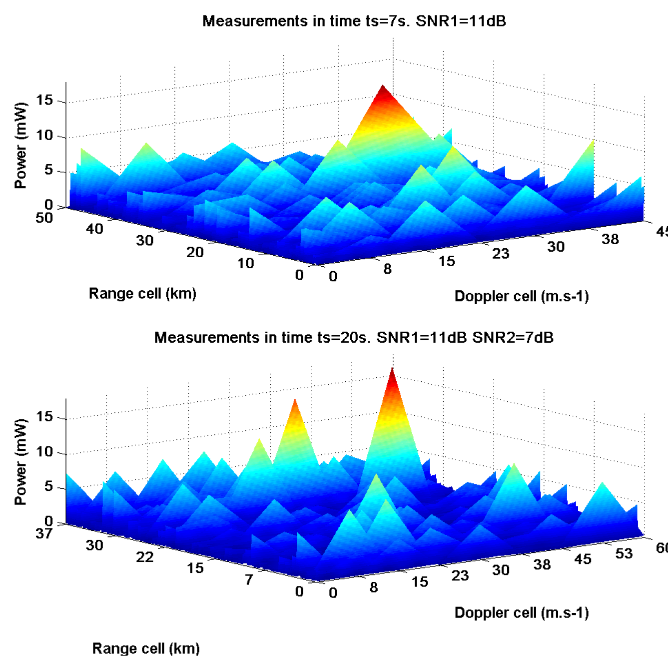

Fig. 1. Measurement data of two targets moving straight toward the sensor with respectively a SNR of 7 dB and 11 dB

B. Results

Simulations are performed for all the setups over $N_{mc} = 40$ Monte Carlo runs. The performances of the different implementations are evaluated by the Root Mean Square Error (RMSE) for each target m at each time step defined by :

$$RMSE(k)^m = \frac{1}{N_{mc}} \sum_{n=1}^{N_{mc}} \|\epsilon_{k,m} - \hat{\epsilon}_{k,m}\|^2$$

where,

$$\epsilon \triangleq \begin{cases} [x_k, y_k], & \text{(a) RMSE in position;} \\ [v_{x_k}, v_{y_k}], & \text{(b) RMSE in velocity;} \\ \rho_k, & \text{(c) RMSE in amplitude.} \end{cases}$$

Figure 2 shows the particle clouds together with the true and the estimated particles state at time steps $k = 7$ and 10 of a simulation given by the first multiple PRFs scheme.

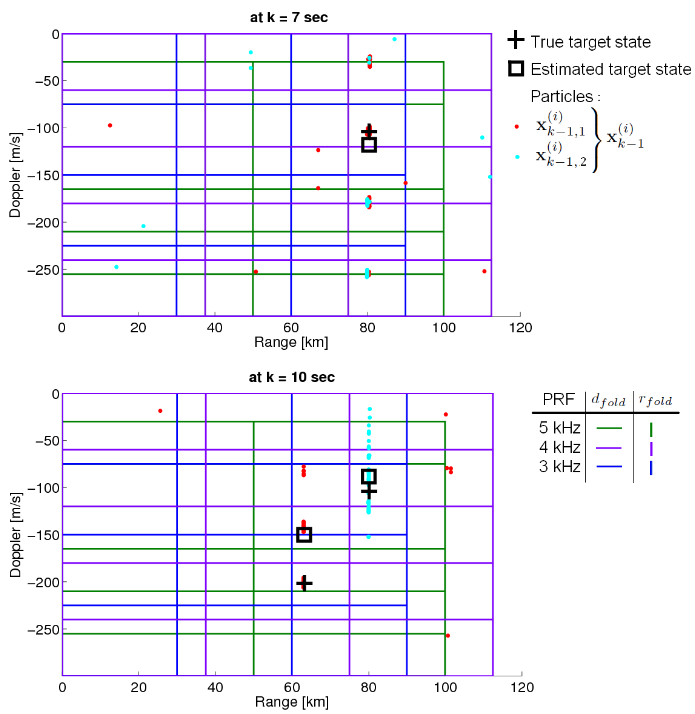

Fig. 2. Particle clouds together with true and estimated state of the both targets at $k = 7$ and 10

Notice that the filter has more trouble to solve the ambiguities in the few first steps following the appearance of the second target with a SNR of 7 dB, specifically for Doppler resolution. Table II illustrate the average RMSE of the different setups.

TABLE II

RMSE ASSOCIATED TO BOTH TARGETS FOR BOTH SCHEMES.

	RMSE	(a) $[m]$	(b) $[m.s^{-1}]$	(c) $[mW]$
Scheme 1	tg1 tg2	61.8 67.7	24.5 48.2	2.3 2.8
Scheme 2	tg1 tg2	58.8 60.7	17.1 17.9	1.2 1.9

After initially having a large estimation error, all filters converge to the true position of the targets.

VI. CONCLUSION

In this paper we presented a modeling setup and an algorithm which can cope with range/doppler ambiguities and eclipsing effects for a multiple target TBD application in radar context. It has been shown that with a fairly straightforward particle filter implementation a good and relatively fast algorithm has been constructed. The proposed method can adapt to any type of waveform and allows you to fully benefit from the frequency agility of the radar. An interesting extension to the presented solution would be to also include clutter effects both in doppler and range. We would also like to combine compare the performance obtained with conventional algorithms and methods, measure the potential gain provided by the proposed Particle Filter.

ACKNOWLEDGMENT

The research leading to these results has received funding from the EU's Seventh Framework Programme under grant agreement n^{o}238710. The research has been carried out in the MC IMPULSE project: https://mcimpulse.isy.liu.se.
The authors would also like to acknowledge Fotios Katsilieris (Thales Nederland B.V.) for providing PRF selection proposal [13].

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