Sensor management for PRF selection in the track-before-detect context

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***Abstract*—We consider the pulse repetition frequency (PRF) selection problem for target tracking in the context of track-before-detect.**

Two sensor management criteria are proposed for solving the PRF selection problem. From the information theoretic class, a Kullback-Leibler divergence based criterion is employed and from the task based class, a criterion that is based on the covariance matrix of the posterior density.

The proposed sensor management criteria succeed in resolving the ambiguities, avoid choosing a PRF that would place the target in a blind zone and they both produce an unexpected but interesting aspect in the results. That is, they can prevent ambiguities from reappearing due to the motion model without being explicitly designed to handle this unpredicted problem.

I. Introduction

The use of PRF during the operation of a radar causes problems such as blind zones, target range and velocity ambiguities that the use of a single PRF cannot resolve [1].

The classical solution to the PRF selection problem is to perform PRF staggering (or PRF jittering) [1], meaning the repetitive (or random) use of at least 3 different PRFs. Even though this approach resolves the range and velocity ambiguities, it has certain disadvantages. During the staggering (or jittering), the target can be placed in a blind zone caused by one or more PRFs. This is undesirable because the corresponding measurement is wasted. Furthermore, these methods are ad-hoc solutions and not optimal in any sense.

In [2], the authors use an evolutionary algorithm that selects 3 out of 8 or 9 PRFs to be transmitted within one dwell time for target search purposes. On the contrary we will try to use one PRF per dwell and solve the ambiguities on dwell to dwell basis for target tracking purposes.

In [3], simulated annealing is applied to obtain a medium PRF (3 out of 8) set by minimizing the range and velocity blind areas in order to achieve an improved blind zone map.

In [4], the maximization of the mutual information is used as a criterion for PRF selection and it is compared to random and coprime PRF selection. We use the maximum expected Kullback-Leibler divergence which is equivalent and we compare it to another adaptive covariance-criterion.

For addressing the PRF selection problem, we look into the target tracking system and its goal, meaning the estimation of the posterior probability density of the target states (position, velocity etc.) conditioned on the received measurements and possibly the extraction and presentation to the operator a proper point estimate [5].

Given the goal of the tracking system, it can be seen that there are two classes of criteria that are directly related to the quantities produced by the tracking system. The information theoretic class which takes into account the probability density of the target states and the task based class which takes into account a derivative quantity, most commonly a point estimate such as the minimum variance estimate. From the first class, we propose employing the maximum expected Kullback-Leibler divergence criterion and from the second class we propose employing the minimum trace of the expected covariance matrix of the posterior density.

The motivation for choosing these two criteria is that we want to find optimal and not ad-hoc solutions to the PRF selection problem. Furthermore, there is an ongoing discussion on whether information theoretic or task-based criteria should be used and what is the practical interpretation of the information theoretic criteria. A more elaborate discussion on the later subject can be found at [6].

The contributions of the approach presented in this paper are:

- a systematic, criterion-based manner to address the PRF selection problem
- the application of two types of sensor management criteria and their implementation by means of a particle filter
- the illustration of the advantages of the proposed criteria by means of a realistic simulated example

The rest of the paper is organized as follows. In section II the problem under consideration is described and in section III the system description is given. The proposed solution is described in section IV and in section V the simulation results are presented. Finally, in section VI the conclusions are discussed and some open questions are also presented.

II. Problem formulation

We consider a scenario where a target has to be tracked by a radar and the radar can utilize several PRFs, of which only one can be used at each time of transmission.

The fact that the radar needs to transmit pulses with a given frequency causes the following problems [1]:

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- blind (range) zones exist where the target cannot be detected. This happens because the radar antenna cannot receive any echoes while transmitting a pulse.
- range ambiguities exist due to the PRF. Assume for example that there have been transmitted n pulses and then the radar starts receiving an echo. How can it be sure from which pulse the echo was received and therefore where exactly the target is?
- velocity ambiguities exist because it is not possible to directly measure the pulse duration difference due to the Doppler effect. For this reason, the phase difference between the transmitted and the received pulses is measured. Obviously, the phase shift is subject to a modulo 2π operation and therefore aliasing can happen.
- conflicting PRF requirements for resolving range and velocity ambiguities. In order to avoid the range ambiguities, low PRFs have to be used but in order to avoid the velocity ambiguities, high PRFs have to be used.

The system under consideration can be mathematically described by the following (discrete time) state and measurement equations:

$$s_k = f(s_{k-1}, w_{k-1}) \tag{1}$$
$$z_k = h(s_k, PRF_k, v_k) \tag{2}$$
$$s_0 \sim p(s_0) \tag{3}$$

where $k = 1, 2, \ldots$ is the time index, $s_k = [x_k \quad v_x \quad y_k \quad v_y \quad \rho_k]^T \in \mathbb{R}^5$ is the 5 dimensional state of the system describing the position and velocity of a target in Cartesian coordinates and the amplitude of its echo, w_k is the 5 dimensional process noise with probability density $p_w(w_k)$, PRF_k is the chosen PRF at time k, $z_k \in \mathbb{R}^3$ is the received radar measurement, meaning the reflected power level of the target in the $N_r \times N_d \times N_b$ sensor cells, N_r, N_d, N_b are the number of range, Doppler, and bearing cells respectively, v_k is the 3 dimensional measurement noise with probability density $p_v(v_k)$, s_0 is the initial state of the system with probability density $p(s_0)$. The vector and possibly non-linear function $f(\cdot) : \mathbb{R}^5 \mapsto \mathbb{R}^5$ describes the dynamics of the system. Similarly, the vector and possibly non-linear function $h(\cdot) : \mathbb{R}^5 \mapsto \mathbb{R}^3$ describes how the measurement z_k is related to the system state s_k and the chosen PRF PRF_k.

The considered problem amounts to finding the optimal, in the sense of the proposed criteria, sequence of PRF_k of the pulses to be transmitted.

The chosen sequence of PRFs will then be used for solving the attached filtering problem of determining the posterior probability density function $p(s_k|Z_k, U_k)$ that describes the kinematic properties and the amplitude of the target. By $Z_k = \{z_1, ..., z_k\}$ the measurement history is denoted and by $U_k = \{PRF_1, ..., PRF_k\}$ the chosen PRF history.

III. System setup

A. Dynamical model

A target with simple dynamics will be considered and therefore a linear Gaussian nearly constant velocity motion model [7] will be employed:

$$s_{k+1} = f(s_k, w_k) = F \cdot s_k + w_k \tag{4}$$

where:

$$w_k \sim \mathcal{N}(\mu, \Sigma)$$

$$F = \begin{bmatrix} 1 & T & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\Sigma = \begin{bmatrix} b_xT^3/3 & b_xT^2/2 & 0 & 0 & 0 \\ b_xT^2/2 & b_xT & 0 & 0 & 0 \\ 0 & 0 & b_yT^3/3 & b_yT^2/2 & 0 \\ 0 & 0 & b_yT^2/2 & b_yT & 0 \\ 0 & 0 & 0 & 0 & b_\rho \end{bmatrix}$$

and $b_x = b_y$ are the power spectral densities of the acceleration noise in the $x - y$ direction, T is the sampling time, $\mu = [0\ 0\ 0\ 0\ 0]^T$ is the mean of the Gaussian noise and b_ρ is the variance of the increment in amplitude.

B. The role of PRF in the radar measurement model

Firstly, the choice of PRF affects the maximum unambiguous range (r_{fold}) and velocity (d_{fold}), see (5). If the range (or velocity) of the target is higher than r_{fold} (or d_{fold}) then the radar cannot be sure what is the correct range (velocity) of the target because any target return from $r + n\frac{c}{2 \cdot PRF}$ would give the same measurement, where $r \in (0, \frac{c}{2 \cdot PRF})$ and $n = 0, 1, 2 \ldots$. A similar relationship holds for the velocity domain.

Secondly, the range and velocity resolution (Δr, Δd) depend on the chosen PRF, the pulse compression factor PCR and the number of transmitted pulses n_P, see (6).

Thirdly, the length of the blind zones (r_{blind}) depends on the pulse width and the location of the blind velocities (d_{blind}) depends on the chosen PRF and the wavelength of the waveform carrier, see (7) where $n = 0, 1, 2, \ldots$.

$$r_{fold} = \frac{c}{2 \cdot PRF} \quad , \quad d_{fold} = \frac{\lambda \cdot PRF}{2} \tag{5}$$
$$\Delta r = PCR\frac{c \cdot PW}{2} \quad , \quad \Delta d = \frac{\lambda \cdot PRF}{2 \cdot n_P} \tag{6}$$
$$r_{blind} = PW \cdot c \quad , \quad d_{blind} = n\frac{\lambda \cdot PRF}{2} \tag{7}$$

By using the equations for r_{fold} and r_{blind} we can derive an expression for the blind zones where the target cannot be detected:

$$r_k \in \left[n\frac{c}{2PRF_k}, \quad n\frac{c}{2PRF_k} + PW \cdot c\right], n = 0, 1, 2 \ldots \tag{8}$$

where r_k is the distance between the radar and the target at time k.

C. Measurement model

The considered application deals with tracking a target in the track-before-detect context. This means that the received measurements are not thresholded in order to obtain plot measurements. On the contrary, all the $N_r \times N_d \times N_b$ sensor cells are considered. We will follow the approach presented in [8] with the difference that in the considered scenario there is only one target and no target birth or death.

In each cell, the measurement will be:

$$\begin{aligned} z_k^{ijl}(s_k, PRF_k) &= |z_{A,k}^{ijl}(s_k, PRF_k)|^2 \\ &= |A_k h_A(s_k, PRF_k) + v_k|^2 \end{aligned} \tag{9}$$

where $z_{A,k}^{ijl}(s_k, PRF_k)$ is the complex amplitude data of the target in the cell ijl, $A_k = \rho_k e^{i\varphi_k}$ is the complex amplitude of the target, $\varphi \in (0, 2\pi)$, $h_A(s_k, PRF_k)$ is the reflection form and v_k is complex Gaussian noise with zero mean and covariance σ^2.

The reflection form $h_A(s_k, PRF_k)$ is given by:

$$h_A(s_k, PRF_k) = e^{-\frac{(r_i - r_k)^2}{2R} - \frac{(d_j - d_k)^2}{2D} - \frac{(b_l - b_k)^2}{2B}} \tag{10}$$

where $i = 1, \ldots, N_r$, $j = 1, \ldots, N_d$, $l = 1, \ldots, N_b$, $R = (\Delta r_k)^2, D = (\Delta d_k)^2, B = (\Delta b_k)^2$ are constants related to the size of a range, a Doppler and a bearing cell respectively. $\Delta r_k, \Delta d_k, \Delta b_k$ are the range, Doppler and bearing resolutions of the radar and

$$r_k = \sqrt{x_k^2 + y_k^2} \left(mod \quad \frac{c}{2 \cdot PRF_k} \right) \tag{11}$$

$$d_k = \dot{r}_k = \frac{x_k v_x + y_k v_y}{\sqrt{x_k^2 + y_k^2}} \left(mod \quad \frac{\lambda \cdot PRF_k}{2} \right) \tag{12}$$

$$b_k = \arctan(y_k / x_k) \tag{13}$$

are the apparent target range and Doppler and its bearing, where c is the speed of light and λ is the wavelength of the waveform carrier.

These measurements, conditioned on the states s_k of the target, are assumed to be exponentially distributed and therefore the likelihood function $p(z_k^{ijl} | s_k, PRF_k)$ will be:

$$p(z_k^{ijl} | s_k, PRF_k) = \frac{1}{\mu^{ijl}} \cdot e^{-\frac{1}{\mu^{ijl}} z_k^{ijl}(s_k, PRF_k)} \tag{14}$$

where

$$\begin{aligned} \mu^{ijl} &= E[z_k^{ijl}(s_k, PRF_k)] \\ &= P h_P^{ijl}(s_k, PRF_k) + 2\sigma^2 \end{aligned} \tag{15}$$

with $P = \rho_k^2$ and

$$\begin{aligned} h_P^{ijl}(s_k, PRF_k) &= \left[h_A^{ijl}(s_k, PRF_k) \right]^2 \\ &= e^{-\frac{(r_i - r_k)^2}{R} - \frac{(d_j - d_k)^2}{D} - \frac{(b_l - b_k)^2}{B}} \end{aligned} \tag{16}$$

As it can be noticed from (15,16) and (11,12,13) the received measurement depends both on the target states (position, velocity, amplitude) and on the PRF that is chosen.

Therefore,

$$z_k^{ijk} = \begin{cases} v_k, & \text{if no target in cell } ijk \\ & \text{or (8) is true} \qquad \text{(17a)} \\ h^{ijk}(s_k, PRF_k, v_k), & \text{if target in cell } ijk \\ & \text{and (8) is false} \qquad \text{(17b)} \end{cases}$$

where $h^{ijk}(s_k, PRF_k, v_k)$ is given by (9).

This means that if we are not careful, we might even choose a PRF that puts the target in a blind zone and therefore makes the target undetectable. This is especially important in the track-before-detect context, where the targets usually have low SNR and no measurements should be wasted.

IV. Proposed Solution

We propose solving the described target tracking problem by employing sensor management criteria for choosing the best PRF and the recursive Bayesian estimation theory for recursively estimating the posterior density $p(s_k | Z_k, U_k)$.

A. Recursive Bayesian estimation

In the recursive Bayesian estimation context, given a probability density function $p(s_{k-1} | Z_{k-1}, U_{k-1})$, the prediction step is performed using the Chapman-Kolmogorov equation:

$$p(s_k | Z_{k-1}, U_{k-1}) = \int p(s_k | s_{k-1}) \cdot p(s_{k-1} | Z_{k-1}, U_{k-1}) \, ds_{k-1} \tag{18}$$

where $p(s_k | s_{k-1})$ is usually determined by the kinematics model of the target.

Then the predictive density $p(s_k | Z_{k-1}, U_{k-1})$ is updated with the received measurement z_k using Bayes' rule

$$p(s_k | Z_k, U_k) = \frac{p(z_k | s_k, PRF_k) \cdot p(s_k | Z_{k-1}, U_{k-1})}{p(z_k | Z_{k-1}, U_k)} \tag{19}$$

where $p(z_k | s_k, PRF_k)$ is the likelihood function and

$$p(z_k | Z_{k-1}, U_k) = \int p(z_k | s_k, PRF_k) \cdot p(s_k | Z_{k-1}, U_{k-1}) \, ds_k \tag{20}$$

is a normalizing constant which in practice does not have to be calculated if we employ a particle filter. Therefore, it will hold that

$$p(s_k | Z_k, U_k) \propto p(z_k | s_k, PRF_k) \cdot p(s_k | Z_{k-1}, U_{k-1}) \tag{21}$$

and (18,19) can be easily approximated using a standard SIR particle filter [5] with N particles s_k^i and corresponding weights q_k^i:

$$\{s_k^i, q_k^i\}, \; i = 1, ..., N \tag{22}$$

such that the approximation converges to the true posterior distribution $p(s_k | Z_k, U_k)$ as $N \to \infty$, see [9].

B. Sensor management criteria

As it is discussed in the introduction, criteria from two classes will be used. From the information theoretic class, the maximum expected Kullback-Leibler divergence will be used. From the task-based class, the minimum trace of the expected covariance matrix of the posterior density will be used.

1) Maximum expected Kullback-Leibler divergence: The maximum expected Kullback-Leibler divergence and the minimum conditional entropy are theoretically equivalent for sensor management purposes [6] but the KL-based criterion has lower computational complexity and this is why it is chosen, see the particle approximations in [10] and [11].

The Kullback Leibler divergence between two densities $q(s)$ and $p(s)$ is given by

$$KL[q(s)||p(s)] = \int q(s) \cdot log\left(\frac{q(s)}{p(s)}\right)\, ds \tag{23}$$

As suggested in [10] for example, the maximum expected KL divergence between the predictive and the simulated posterior density can be used for choosing the best PRF PRF_k. The sensor management criterion would then be:

$$PRF_k = \arg\max_{PRF}\; E_Z\left\{KL[q(s)||p(s)]\right\} \tag{24}$$

where

$$q(s) = p(s_k|z_k, Z_{k-1}, PRF, U_{k-1}) \tag{25}$$
$$p(s) = p(s_k|Z_{k-1}, U_{k-1}) \tag{26}$$

and z_k is the simulated measurement using s_k and PRF.

We use a particle approximation of the expected KL divergence similar to [10]. In the following formulas, z_k^p will denote the simulated measurement at time k, using s_k^p with weight q_{k-1}^p and PRF_k.

$$\begin{aligned} &E_Z[KL(p(s_k|z_k, Z_{k-1}, PRF_k, U_{k-1})||p(s_k|Z_{k-1}, U_{k-1}))] \\ &\;= \int p(z_k|s_k, PRF_k) \cdot \int p(s_k|Z_{k-1}, U_{k-1}) \cdot \\ &\quad \cdot log\left(\frac{p(z_k|s_k, PRF_k)}{p(z_k|Z_{k-1}, U_{k-1}, PRF_k)}\right)\, ds_k\; dz_k \\ &\;\approx \sum_{p=1}^{P} q_{k-1}^p \left\{ log\left(\frac{p(z_k^p|s_k^p, PRF_k)}{\hat{p}_M(z_k^p|Z_{k-1}, U_{k-1}, PRF_k)}\right)\right\} \end{aligned} \tag{27}$$

where

$$\hat{p}_M(z_k^p|Z_{k-1}, U_{k-1}, PRF_k) = \sum_{m=1}^{M}\left\{q_{k-1}^m \cdot p(z_k^p|s_k^m, PRF_k)\right\} \tag{28}$$

This evaluation is repeated K times (1 time per PRF) and then the PRF that gives the highest value in (27) is chosen.

In (27) and (28), M denotes the number of particles used within the criterion and P is the number of simulated measurements.

The computational complexity of this criterion is $\mathcal{O}(MPK)$ and the corresponding computational complexity of the equivalent conditional entropy would be $\mathcal{O}(M^2PK)$.

2) Minimum trace of the expected covariance matrix: If we assume that the uncertainty about the target's attributes (position, velocity and amplitude) is sufficiently represented by the mean and the covariance of the corresponding probability density function, then it is intuitive to choose a criterion that selects the PRF which leads to the minimum trace of the expected covariance matrix of the posterior density.

The covariance of a probability density function $p(s)$ of a random variable S is given by:

$$Cov[p(s)] = \int_s (s-\mu_s)(s-\mu_s)^T p(s)\; ds \tag{29}$$

where $\mu_s = \int s \cdot p(s)ds$ is the expected value of S.

The corresponding sensor management criterion would then be:

$$PRF_k = \arg\min_{PRF} \text{tr}[E_Z\left\{Cov[p(s_k|z_k, Z_{k-1}, PRF, U_{k-1})]\right]\} \tag{30}$$

We approximate (30) as follows:

$$\begin{aligned} &E_Z[Cov(p(s_k|z_k, Z_{k-1}, PRF_k, U_{k-1}))] \\ &\quad = \int p(z_k|Z_{k-1}, PRF_k, U_{k-1}) \cdot \int (s_k - \mu_{s_k})(s_k - \mu_{s_k})^T \\ &\qquad\quad \cdot p(s_k|z_k, Z_{k-1}, PRF_k, U_{k-1})\; ds_k\; dz_k \\ &\quad \approx \sum_{p=1}^{P} q_{k-1}^p \left\{\sum_{m=1}^{M} q_{k-1}^m (s_k^m - \mu_{s_k}^p)(s_k^m - \mu_{s_k}^p)^T\right\} \end{aligned} \tag{31}$$

where

$$\begin{aligned} \mu_{s_k}^p &= \int s_k \cdot p(s_k|z_k^p, Z_{k-1}, PRF_k, U_{k-1})\; ds_k \\ &\approx \sum_{n=1}^{M} q_k^m \cdot s_k^m \end{aligned} \tag{32}$$

$$s_k^m \sim p(s_k|z_k^p, Z_{k-1}, PRF, U_{k-1}) \tag{33}$$

which is evaluated using z_k^p in (18) and (19). We remind to the reader that z_k^p denotes the simulated measurement at time k, using s_k^p with weight q_{k-1}^p and PRF_k. The updated weight q_k^m is evaluated using the simulated measurement z_k^p in (14) followed by a normalization step.

Again, this evaluation is repeated K times (1 time per PRF) and then we choose the PRF that gives the lowest value in (30). In (31) and (32), M denotes the number of particles used within the criterion and P is the number of simulated measurements.

The computational complexity of the covariance based criterion is $\mathcal{O}[(M+1)PK]$ which is between $\mathcal{O}(MPK)$ and $\mathcal{O}(M^2PK)$.

TABLE I
THE CHOSEN PRFS, PULSE WIDTHS (PW), PULSE COMPRESSION FACTORS (PCR) AND NUMBER OF PULSES (n_P).

PRF kHz	**PW** $sec \cdot 10^{-6}$	**PCR**	n_P
1.4	53	0.013	3
4	18.9	0.036	8
5	15.1	0.045	10
5.5	13.7	0.05	11
23.5	3.2	0.21	47

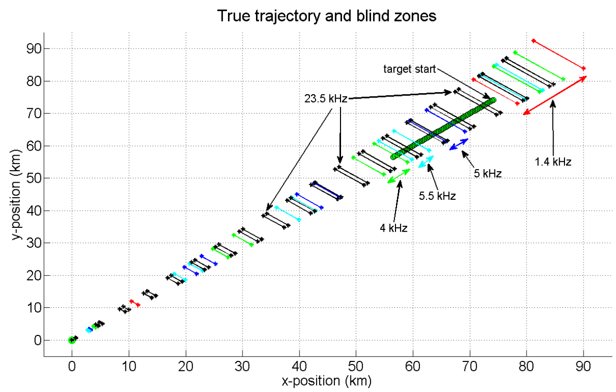

Fig. 1. The scenario considered in our simulations.

V. SIMULATIONS

In the simulated scenario, the radar is assumed to be at the origin of the axes. The target to be tracked starts at $k = 0$ from $[x_0^{true}, y_0^{true}] = [74.2, 74.2]\ km$ and moves with constant velocities $v_x^{true} = v_y^{true} = -300\ m/s$ for 60 sec towards the radar. Its SNR is assumed to be 11 dB.

The chosen PRFs along with the corresponding pulse widths (PW), pulse compression factors (PCR) and number of transmitted pulses (n_P) are shown in Table I. They were chosen such that the range and velocity resolutions and the duty cycle ($PRF \cdot PW$) are constant. These conditions make sure that no PRF is favored due to better resolution or more covered area.

Fig. 1 depicts the scenario under consideration in our simulations. In Fig. 1 the position of the radar, the trajectory of the target and the blind zones caused by each PRF can be seen.

The $N = 10^4$ particles are initially distributed uniformly such that:

- $r_0 \in [0, 115]\ km$, $b_0 \in [0.75, 0.85]\ rad$, $d_0 \in [-500, 0]$ m/s
- $SNR_0 \in [4, 16]\ dB$
- $\rho_0 = \sqrt{2\sigma^2 \cdot 10^{SNR/10}} \in [1.5849, 6.3096]\ Watts$
- $\varphi_{0:k}$ is considered random and does not affect the results

We assume that we want to track a highly maneuverable target, such as a fighter or a missile, and therefore we use high process noise. For the dynamical model we use:

- $b_x = b_y = 400\ m^2/s^4$ and $b_\rho = 10^{-3}\ Watts^2$
- $T = 1\ sec$ and $k = 1, \ldots, 60\ sec$

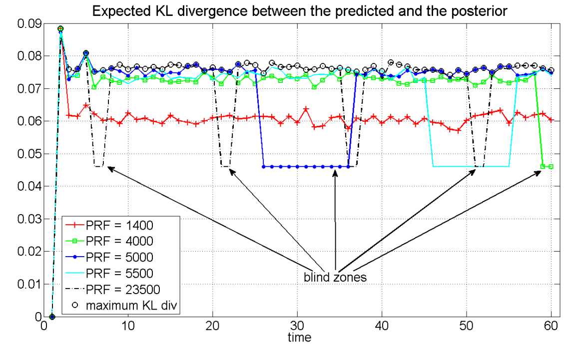

Fig. 2. The expected KL divergence between the predictive and the posterior density for each PRF. The PRFs that put the target in a blind zone result in a lower KL divergence and therefore they are not chosen.

The parameters for the measurement model are:

- $\lambda = 0.03\ m$, $c = 3 \cdot 10^8\ m/s$ and $\sigma^2 = 1/2$
- beam width $\Delta b = 0.1\ rad \simeq 5.7^o$
- $\Delta r, \Delta d$ according to (11, 12) and Table I

Due to the high computational load involved in our experiments, we only choose 100 out of the $N = 10 \cdot 10^3$ particles and we simulate 1 measurement per chosen particle for the evaluation of the criteria, meaning 100 measurements in total. The choice of the 100 particles is performed by a multinomial resampling step. This procedure has to be repeated 5 times because we employ 5 different PRFs. According to the notation in [10], we use $M = 100$ particles, $P = 100$ simulated measurements (1 from each particle) and $K = 5$ (5 PRFs) for evaluating the criteria.

Fig. 2 and 3 show a characteristic example of the obtained sensor management results for the two criteria. In Fig. 2, higher KL divergence represents better PRF choice and therefore, the PRFs that would put the target in a blind zone are avoided because they lead to lower KL divergence. On the contrary, in Fig. 3, lower trace of the covariance matrix represents better PRF choice and therefore, the PRFs that would put the target in a blind zone are avoided because they lead to higher trace of the covariance matrix.

Fig. 4 and 5 show the sequence of the chosen PRFs produced by the two criteria. It can be noticed that the highest PRF is preferred. This can be explained by the fact that a high process noise is used for tracking a highly maneuverable target. This leads to ambiguities being created at every time instance in the velocity domain and therefore the highest PRF must be chosen for resolving them. The aforementioned explanation was verified by a new set of experiments with lower process noise where the medium PRFs were also chosen, provided that they would not place the target in a blind zone.

VI. CONCLUSIONS

The proposed criteria were found to produce similar results in the sense that both criteria resolve the ambiguities and avoid choosing a PRF that would place the target in a blind zone.

Furthermore, both criteria produced an unexpected but interesting result, namely the highest PRF would be preferred

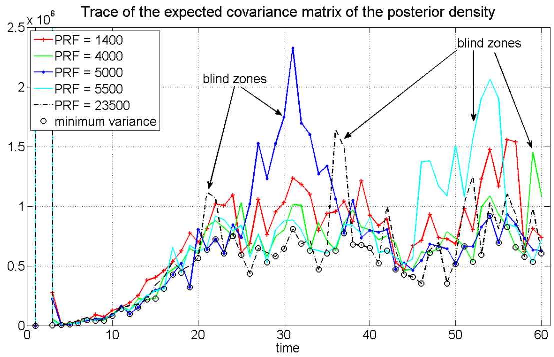

Fig. 3. The trace of the expected covariance matrix of the posterior density for each PRF. The PRFs that put the target in a blind zone result in a higher covariance and therefore they are not chosen.

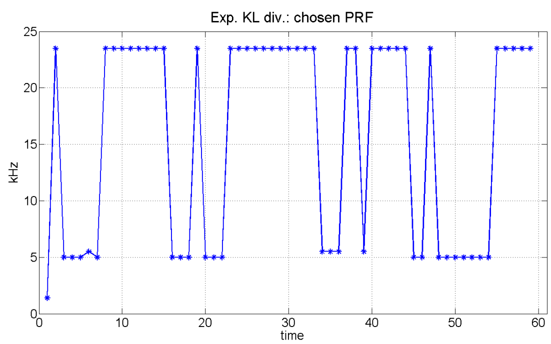

Fig. 4. The sequence of the chosen PRFs by KL-based criterion. Notice that the highest PRF is preferred.

by both criteria unless its selection would lead to placing the target in a blind zone. The selection of the high PRF depends on the maneuverability of the target to be tracked, as verified by a second series of experiments where tracking targets with lower maneuverability did not lead to high PRF preference.

The fact that both criteria managed to detect that the motion model creates problems in the tracking process by constantly introducing velocity ambiguities and they managed to tackle this problem by choosing the highest PRF when necessary is an extra advantage over the classical solutions. PRF staggering would address this problem every $5\ sec$, when the lowest PRF would be used, and PRF jittering would address it at random time instances.

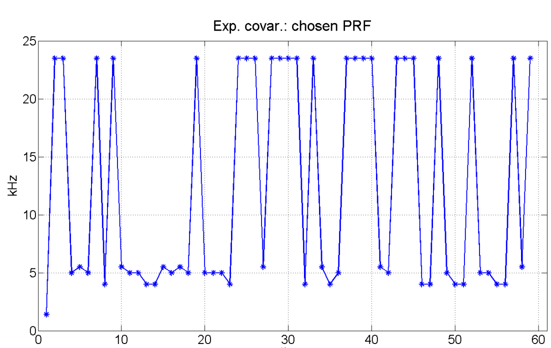

Fig. 5. The sequence of the chosen PRFs by the covariance-based criterion. Notice that the highest PRF is preferred.

We would also like to point out that the obtained results are also applicable when plot measurements are used instead of the unthresholded measurements.

An interesting extension to the presented solution would be to also include clutter effects both in Doppler and range. We would also like to explore the behavior of the criteria in a maneuvering and/or multi-target scenario.

Another interesting and partially open question is to explore how the results obtained by the KL based criterion perform in the context of the covariance based criterion and vice versa. According to the discussion in [11] about the representation of uncertainty in unimodal distributions, we expect the aforementioned comparison to indicate that the criteria produce very similar results. A more practical explanation is that the criteria have similar behaviors because both resolve the ambiguities and try to avoid the blind zones.

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