Fig. 7. Position RMSE over time for the standard *Unconstrained PF* and the *Pseudo-Measurements IMM-PF*. The filter does not overcome the increase in error due to the constraint violation for $k \in [21, 40]$.
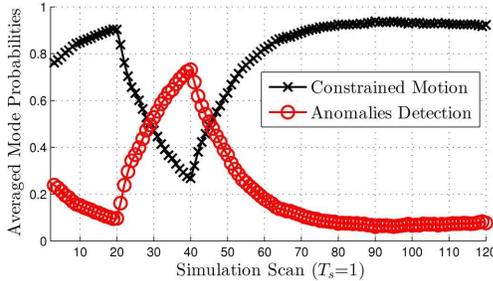

Fig. 8. Average Posterior Mode Probabilities for the *IMM-PF*. Notice how the filter is able to detect the constraint violation occurring for $k \in [21, 40]$.

to what is shown in figs. 4 and 5. However, from fig. 8 we notice that the filter is able to detect the constraint violation.

VII. CONCLUSION

This paper addresses fundamental issues arising when tracking a target that is subject to known constraints. The Particle Filter is the workhorse for such cases. If constraints are known and correctly modeled, then the PF converges to the correct a posteriori PDF. In particular, we formally showed that in such case, processing of external knowledge in the prediction or in the update step of the filtering recursion are equivalent from a Bayesian viewpoint. This means that we can use the *Pseudo-Measurements* technique for optimal processing of external knowledge. The equivalence is also shown from a practical viewpoint through a simple 2D tracking example. Here a particle approximation of the Kullback-Leibler Divergence is used as a measure of closeness between empirical densities.

We then focused on the case of *soft constraints*, and demonstrated through simulations that if unmodeled behaviors occur, the performance degradation is important. Such problem cannot be solved in general without focusing on a specific application. In particular, we showed that when the *detection of unmodeled behaviors* is of interest, then an IMM-PF with suitably chosen models effectively solves the problem.

VIII. ACKNOWLEDGMENT

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