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Group Object Tracking with a Sequential Monte Carlo Method Based on a Parameterised Likelihood Function

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Abstract. Group objects are characterised with multiple measurements originating from different locations of the targets constituting the group. This paper presents a novel Sequential Monte Carlo approach for tracking groups with a large number of components, applicable to various nonlinear problems. The novelty in this work is in the derivation of the likelihood function for nonlinear measurement functions, with sets of measurements belonging to a bounded spatial region. Simulation results are presented when a group of 50 objects is surrounded by a circular region. Estimation results are given for the group object center and extent.

Keywords. sequential Monte Carlo methods, measurement uncertainty, nonlinear estimation, group object tracking.

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1 Motivation

Group object tracking is concerned with finding the patterns of behaviour and tracks of a whole group instead of each object separately. The group of objects are generating multiple measurements which origin is unknown. Different methods are proposed in the literature for solving the problem of group object tracking, e.g., [2, 9, 10]. Several approaches consist in estimating the extent of the group, e.g., with random matrices as suggested in [3,9]. Other related works are [4,5,8,11–14].

In general the measurement uncertainty can belong to a hypercube or to another spatial shape. In our approach, we consider the general case with a nonlinear measurement equation and measurements. The main contributions of the work is in the derived likelihood function based on a parameterised shape and in the

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developed Sequential Monte Carlo filter for group objects. Then we propagate this spatial measurement uncertainty through the Bayesian estimation framework.

In this work inspired by ideas from [7, 8] we developed a novel approach for tracking groups of objects consisting of a large number of components. The remaining part of this paper is structured as follows. Section 2 formulates the problem. Section 3 derives the measurement likelihood function. Section 4 presents simulation results about the performance of the proposed approach and finally Section 5 summarises the outcomes.

2 Group Object Tracking Within the Sequential Monte Carlo Framework

The problem of state estimation of groups of objects is considered within the Sequential Monte Carlo framework. Each group, $g = 1, ..., n_g$, is comprised of n_T^g targets. The group state vector is defined as $\boldsymbol{X}_k^g = \left(\boldsymbol{x}'_{g,k}, \boldsymbol{G}'_k\right)'$, where $\boldsymbol{x}_{g,k}$ is the vector containing the position and speed coordinates of the center of the region surrounding the group g, at time k, $\boldsymbol{G}_k$ is a vector characterising the group and $'$ is the transpose operator. The groups of objects give rise to a set of measurements $\boldsymbol{Z}_k = \{\boldsymbol{z}_{m,k}\}_{m=1}^{M_k}$. The goal is to estimate $\boldsymbol{X}_k^g$ given the measurements $\boldsymbol{Z}_{1:k} = \{\boldsymbol{Z}_1, ..., \boldsymbol{Z}_k\}$, collected up to time k. Within the sequential Bayesian framework this goal can be achieved by constructing the group state probability density function (pdf) based on the incoming sensor data. According to the Bayes' rule the filtering pdf $p(\boldsymbol{X}_k^g|\boldsymbol{Z}_{1:k})$ of the state vector $\boldsymbol{X}_k^g$ given a sequence of sensor measurements $\boldsymbol{Z}_{1:k}$ can be written as

$$p(\boldsymbol{X}_k^g|\boldsymbol{Z}_{1:k}) = \frac{p(\boldsymbol{Z}_k|\boldsymbol{X}_k^g)p(\boldsymbol{X}_k^g|\boldsymbol{Z}_{1:k-1})}{p(\boldsymbol{Z}_k|\boldsymbol{Z}_{1:k-1})}, \tag{2.1}$$

where $p(\boldsymbol{Z}_k|\boldsymbol{Z}_{1:k-1})$ is the normalising constant. The state *predictive* distribution is given by the equation

$$p(\boldsymbol{X}_k^g|\boldsymbol{Z}_{1:k-1}) = \int p(\boldsymbol{X}_k^g|\boldsymbol{X}_{k-1}^g)p(\boldsymbol{X}_{k-1}^g|\boldsymbol{Z}_{1:k-1})d\boldsymbol{X}_{k-1}^g. \tag{2.2}$$

The evaluation of the right hand side of (2.1) involves integration which can be performed within the Sequential Monte Carlo framework (known also as Particle Filtering) [1] by approximating the posterior pdf $p(\boldsymbol{X}_k^g|\boldsymbol{Z}_{1:k})$ with a set of particles $\boldsymbol{X}_k^{g(i)}$, $i = 1, \ldots, N$ and their corresponding weights $w_k^{(i)}$. Then the posterior density function can be written as follows

$$p(\boldsymbol{X}_k^g|\boldsymbol{Z}_{1:k}) \approx \sum_{i=1}^{N} w_k^{(i)} \delta(\boldsymbol{X}_k^g - \boldsymbol{X}_k^{g(i)}), \tag{2.3}$$

where $\delta(\cdot)$ is the Dirac delta function, and the weights are normalised such that $\sum_i w_k^{(i)} = 1$.

Each pair $\{\boldsymbol{X}_k^{g(i)}, w_k^{(i)}\}$ characterises the belief that the group g is in state $\boldsymbol{X}_k^{g(i)}$. Note that, for example in the case of a circle, we assume that $\boldsymbol{X}_k^{g(i)} = (x_{c,k}, y_{c,k}, \dot{x}_{c,k}, \dot{y}_{c,k}, r_k)'$, where $x_{c,k}, y_{c,k}$ are the center position coordinates, $\dot{x}_{c,k}$, $\dot{y}_{c,k}$ are the respective speed components and r_k is the radius. *In a general case a particle* $\boldsymbol{X}_k^{g(i)}$ *characterises a spatial shape.* An estimate of the variable of interest is obtained by the weighted sum of the particles. Two major stages can be distinguished: *prediction* and *update*. During prediction, each particle is modified according to the state evolution model, including the addition of random noise. In the measurement update stage, the weight are re-evaluated using a likelihood term. The residual resampling [15] is added to prevent sample impoverishment.

3 Measurement Likelihood for Group Object Tracking

In the context of group object tracking, the prediction step is generally well studied for various classes of interval and spatial representations of uncertainties. Ellipsoids, spheres and polytope families can be easily propagated when the system dynamics and sensor models are linear.

The update step with the likelihood calculation is less studied or often studied with a restriction to a particular class of sets or a particular type of measurement models. The aim of this paper is to derive general likelihood calculation procedures based on Monte Carlo methods without a restriction on the type of regions of interest or the type of measurement available. For that purpose, the main idea of this paper is to introduce a sampling step for regions of interest in the groups regions. This sampling aims to represent the probability of a given point, in the state space, to be the origin of a measurement. This section describes in detail the newly proposed method. As an illustration, the case of a group object with circular form is considered.

The sensor state of sensor s at time k is denoted by $\boldsymbol{x}_{s,k}$. Assume that at each measurement time k the groups generate a matrix $\boldsymbol{Z}_k$ of $M_k = \sum_{g=1}^{G} \sum_{t=1}^{n_{T_g}} n_T^g M_{t,k}$ measurement vectors. The number of measurements $M_{k,t}$ originating from the target t is considered to be Poisson-distributed random variable with mean value of μ_t, i.e. $M_{t,k} \sim Poisson(\mu_t)$. The measurements originating from a target t, having a state vector $\boldsymbol{x}_{t,k}^g$ are assumed to be conditionally independent, i.e.

$$p(\boldsymbol{Z}_k | \boldsymbol{x}_{t,k}) = \prod_{m=1}^{M_{t,k}} p(\boldsymbol{z}_{m,k} | \boldsymbol{x}_{t,k}^g). \tag{3.1}$$

For each target t, the generation of measurements over the observation space at time step k is modelled as a nonhomogeneos Poisson point process (PPP). Let $\lambda_t(\boldsymbol{z}_{m,k}|\boldsymbol{x}_{t,k}, \boldsymbol{G}_k)$ be the intensity of the PPP. The pdf of each measurement $\boldsymbol{z}_m$, produced in a region A of the observation space (sensor field-of-view) can be expressed by $p(\boldsymbol{z}_{m,k}|\boldsymbol{x}^g_{t,k}, \boldsymbol{G}_k) = \lambda_t(\boldsymbol{z}_{m,k}|\boldsymbol{x}^g_{t,k}, \boldsymbol{G}_k)/\mu_t(A)$, where $\mu_t(A)$ is the expected number of measurements falling in A. We assume that $\mu_t(A)$ is known. It is assumed also that the measurements received over the observation volume result from the superposition of n^g_T conditionally independent nonhomogeneous PPPs. Since the superposition of such independent Poisson processes is a Poisson process with parameter that is $\sum_{t=1}^{n^g_t} \lambda_t$, where the expected total number of measurements, received in the time instant k is given by $\mu(A) = \sum_{t=1}^{n^g_t} \mu_t(A)$. Similarly to [7], the joint likelihood of the number n^g_T of measurement falling in the observation volume A is given by

$$p(\boldsymbol{Z}_k|\boldsymbol{X}^g_k) \approx \prod_{m=1}^{M_k} \sum_{t=1}^{n^g_T} \mu_t(A) p(\boldsymbol{z}_{m,k}|\boldsymbol{x}^g_{t,k}). \tag{3.2}$$

Note that in this work the state vector $\boldsymbol{x}^g_{t,k}$ of each target is not estimated. Instead the proposed technique estimates the center of the region and its extent.

3.1 Introduction of the Notion of the Visible Surface

The measurement likelihood $p(\boldsymbol{z}_{m,k}|\boldsymbol{X}^g_k)$ of the group target can be represented with the relation

$$p(\boldsymbol{z}_{m,k}|\boldsymbol{X}^g_k) = \int\limits_{\mathbb{R}^{n_V}} p(\boldsymbol{z}_{m,k}|\boldsymbol{V}^g_k) p(\boldsymbol{V}^g_k|\boldsymbol{X}^g_k) d\boldsymbol{V}^g_k, \tag{3.3}$$

where $\boldsymbol{V}^g_k \in \mathbb{R}^{n_v}$ denotes a source of measurement in the state space. In practice, these visible sources $\boldsymbol{V}^g_k$ depend on the group state vector $\boldsymbol{X}^g_k$ and on the sensor state (e.g., the sensor position and angle of view). The pdf $p(\boldsymbol{V}^g_k|\boldsymbol{X}^g_k)$ represents the probability of a point in the state space to be a source of measurement given the group $\boldsymbol{X}^g_k$. The surface of the group with state $\boldsymbol{X}^g_k$ visible from the sensor with a state vector $\boldsymbol{x}_{s,k}$ is denoted by $\mathcal{V}_k(\boldsymbol{X}^g_k, \boldsymbol{x}_{s,k})$.

We assume that the sources of the measurements at time step k, given the target state and the sensor prior state are uniformly distributed along the region $\mathcal{V}_k(\boldsymbol{X}^g_k, \boldsymbol{x}_{s,k})$, visible from the sensor $\boldsymbol{x}_{s,k}$, i.e.

$$p(\boldsymbol{V}^g_k|\boldsymbol{X}^g_k) = p(\boldsymbol{V}^g_k|\boldsymbol{X}^g_k, \boldsymbol{x}_{s,k}) = \mathcal{U}_{\mathcal{V}_k(\boldsymbol{X}^g_k, \boldsymbol{x}_{s,k})}(\boldsymbol{V}^g_k), \tag{3.4}$$

where $\mathcal{U}_{\mathcal{V}_k(\boldsymbol{X}^g_k, \boldsymbol{x}_{s,k})}(\cdot)$ represents the uniform pdf in $\mathcal{V}_k(\boldsymbol{X}^g_k, \boldsymbol{x}_{s,k})$.

Inserting (3.4) into (3.3) gives

$$p(\boldsymbol{z}_{m,k}|\boldsymbol{X}_k^g) = \int\limits_{\mathbb{R}^{n_V}} p(\boldsymbol{z}_{m,k}|\boldsymbol{V}_k^g)\mathcal{U}_{\mathcal{V}_k(\boldsymbol{X}_k^g,\boldsymbol{x}_{s,k})}(\boldsymbol{V}_k^g)d\boldsymbol{V}_k^g. \tag{3.5}$$

3.2 Parametrisation of the Visible Surface

The likelihood (3.3) can be calculated using the following Monte Carlo integration:

$$\begin{aligned} p(\boldsymbol{z}_{m,k}|\boldsymbol{X}_{k|k-1}^{g(i)}) &= \int\limits_{\mathbb{R}^{n_x}} p(\boldsymbol{z}_{m,k}|\boldsymbol{V}_k^g)p(\boldsymbol{V}_k^g|\boldsymbol{X}_{k|k-1}^{g(i)})d\boldsymbol{V}_k^g \\ &\approx \sum_{\ell=1}^{S} p(\boldsymbol{z}_{m,k}|\boldsymbol{V}_k^{g(\ell)})p(\boldsymbol{V}_k^{g(\ell)}|\boldsymbol{X}_{k|k-1}^{g(i)}). \end{aligned} \tag{3.6}$$

After the prediction step, a set of weighted particles $\{(\boldsymbol{X}_{k|k-1}^{g(i)}, w_{k|k-1}^{(i)})\}_{i=1}^N$ is available. First, the visible area $\mathcal{V}_k(\boldsymbol{X}_{k|k-1}^{g(i)}, \boldsymbol{x}_{s,k})$ is determined. Then for each particle $\boldsymbol{X}_{k|k-1}^{g(i)}$ the likelihood function $p(\boldsymbol{V}_k^{g(\ell)}|\boldsymbol{X}_{k|k-1}^{g(i)})$ is defined from $\mathcal{V}_k(\boldsymbol{X}_{k|k-1}^{g(i)}, \boldsymbol{x}_{s,k})$, with $\ell = 1, ..., S$, where $S = QN$ and Q is the number of point samples generated uniformly inside each of the N circle-particles, characterising the group extent.

The term $p(\boldsymbol{z}_{m,k}|\boldsymbol{V}_k^{g(\ell)})$ in (3.6) can be easily calculated in a classical way depending on the problem, for example using a Gaussian measurement noise distribution

$$p(\boldsymbol{z}_{m,k}|\boldsymbol{V}_k^{g(\ell)}) = \frac{1}{\sqrt{2\pi||\boldsymbol{R}||}} e^{-\frac{(\boldsymbol{z}_{m,k}-\boldsymbol{z}_k^{(\ell)})\boldsymbol{R}^{-1}(\boldsymbol{z}_{m,k}-\boldsymbol{z}_k^{(\ell)})^T}{2}}, \tag{3.7}$$

where $\boldsymbol{R}$ is a known measurement noise covariance matrix.

A simple expression of $p(\boldsymbol{V}_k^{g(\ell)}|\boldsymbol{X}_{k|k-1}^{g(i)})$, in the case of circular group shape, can be

$$p(\boldsymbol{V}_k^{g(\ell)}|\boldsymbol{X}_{k|k-1}^{g(i)}) = \mathcal{U}_{\mathcal{C}(x_{c,k}^{(i)}, y_{c,k}^{(i)}, r_{k|k-1}^{(i)})}\left(x_k^{(\ell)}, y_k^{(\ell)}\right), \tag{3.8}$$

where $\mathcal{C}(x_{c,k}^{(i)}, y_{c,k}^{(i)},)$ is a circle, that has a center coordinates $x_{c,k}^{(i)}, y_{c,k}^{(i)}$ and radius $r_{k|k-1}^{(i)}$ taken from the i^{th} particle $\boldsymbol{X}_{k|k-1}^{g(i)}$. The total number of point samples $\boldsymbol{V}_k^{g(\ell)} = \{x_k^{(\ell)}, y_k^{(\ell)}\}$ for the Monte Carlo integration of (3.6) is given by S. First, uniformly distributed point samples $\boldsymbol{V}_k^{g(\ell)}$ are generated as proposed in [6]. Next, the likelihood values are calculated. These two constitute the basics of calculating (3.6).

4 Performance Evaluation

An example with one group that consists of 50 targets is considered for the duration of 200 time steps. The scenarios are repeated for 20 Monte Carlo iterations. We are estimating the vector $\boldsymbol{X}_k^g = (x_{c,k}, y_{c,k}, \dot{x}_{c,k}, \dot{y}_{c,k}, r_k)'$, with $n_g = 1$, characterising the group. The initial coordinates of the targets are uniformly distributed in a square $50m \times 50m$. A constant velocity model is considered for the target evolution model. The initial position is defined by the mass center of the actual targets positions with added Gaussian noise with standard deviation of $5m$ for both coordinates $x_{c,k=0}$ and $y_{c,k=0}$. The velocity is $\dot{x}_{c,k=0} = \mathcal{N}(100, 2^2)m/s$ and $\dot{x}_{c,k=0} = \mathcal{N}(100, 2^2)m/s$, respectively. The system dynamic noise is characterised by $\sigma_x = \sigma_y = 0.1$.

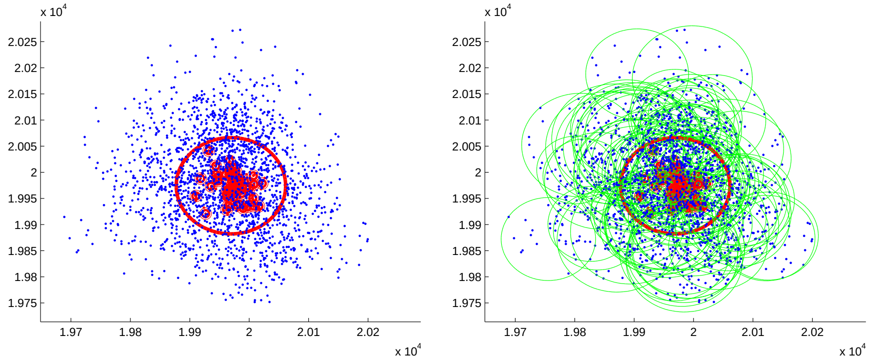
Measurements consisting of range and bearing are considered. They are collected by sensors randomly positioned the and covering the target trajectory. The number of the measurements originating from a target t is Poisson distributed random variable with $\mu_t = 5$. The performance of the algorithm has been evaluated with a variable number of particles N from 20 to 300. For each of these circle-particles the integration of (3.6) is carried out with $Q = 5$ uniformly distributed point samples.

Fig. 1 a) and b) and Fig. 2 a) and b) represent different plots of the same time step for a scenario similar to the one described above, with $N = 100$ circle-particles and $Q = 20$ point samples. Visualisation of the sampling can be seen with blue colour in Fig. 1 a) and b), while the particles are plotted with the green circles in Fig. 1 b). The point samples with their weights are shown in Fig. 2 a). In Fig. 2 b) the real targets can be seen as small red double circles, the measurements are represented as red dots, the estimated group extent is given as a big red circle.

For the purpose of comparison, we define an optimal extent, that is the smallest circle surrounding all the targets within the group. Next, based on the proposed algorithm the group extent is estimated. We form the ratio between these two extents (estimated over optimal). This ratio as well as the number of the targets what belong to the estimated extent is shown in Fig. 3. The first plot shows a convergence to an idealistic circle. On the second plot we see that the percentage of the targets contained within the estimated circle is close to 100% for a sufficient number of particles, $N = 200$ or more.

5 Conclusions

This paper presents a Sequential Monte Carlo approach for group object tracking. The novelty of this work consists in the proposed algorithm for the likelihood estimation. The performance of the algorithm is validated over an example with a



(a) Samples for the Monte Carlo integration of (3.6) - blue dots, estimated extent - big red circle, targets - small red double circles, measurements - red dots

(b) Particles - green circles

Fig. 1: Visualisation of the considered simulation scenario

group having 50 components. We show that the algorithm successfully evaluates the center and the group extent based on measurements with an unknown origin coordinates. The algorithm is scalable and applicable to a large number of components (e.g. hundreds or thousands).

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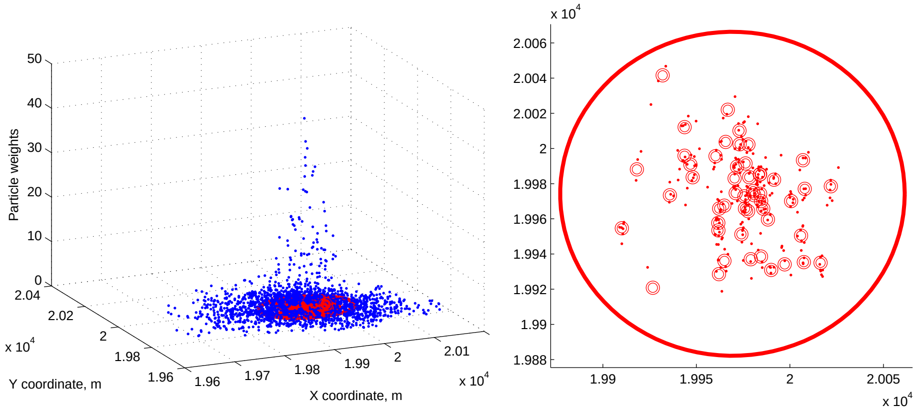

(a) Samples for the Monte Carlo integration of (3.6) with their weights - blue dots

(b) Targets - small red double circles, measurements - red dots, estimated group extent - big red circle

Fig. 2: Visualisation of the considered simulation scenario

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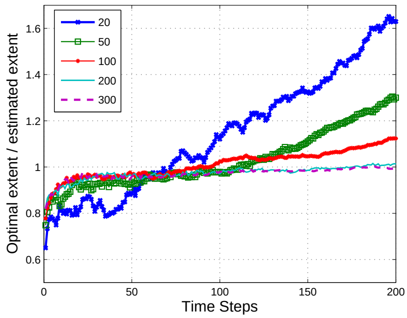
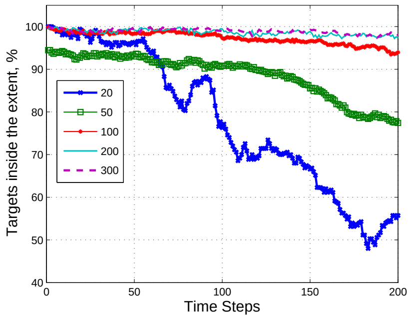


(a) Ratio between the optimal and the estimated group extent



(b) Percentage of targets within the estimated extent

Fig. 3: Simulation results for different number of particles

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