A Sequential Monte Carlo Approach for Extended Object Tracking in the Presence of Clutter

Nikolay Petrov[1], Lyudmila Mihaylova[1], Amadou Gning[1] and Donka Angelova[2]
[1] Lancaster University, UK[2], Bulgarian Academy of Sciences, Bulgaria
Email: {n.petrov, mila.mihaylova, e.gning}@lancaster.ac.uk, donka@bas.bg

Abstract: This paper presents a novel Sequential Monte Carlo (SMC) approach for extended object tracking in the presence of clutter. The problem is formulated for general nonlinear problems. The main contribution of this work is in the derivation of the likelihood function for nonlinear measurement functions, with sets of measurements belonging to a bounded region. Simulation results are presented when the object is surrounded by a circular region. Accurate estimation results are presented both for the object kinematic state and object extent.

1 Motivation

Extended Object Tracking (EOT) is an important application where the interest is in finding estimates of the centre of the area surrounding an object and the object extent/size. The extended object usually leads to multiple measurements. Different methods are proposed in the literature for dealing with this problem. Most of the methods separate the problem of kinematic state estimation from the problem of parameter state estimation such as in [KF09, Koc08]. A comparison between the approach with random matrices and the combined-set theoretic approach is presented in [BFF$^+$10]. An approach with SMC method for EOT is proposed in [VIG05]. Other related works are [BH09a, BH09b, NKPH10].

In general the measurement uncertainty can belong to a hypercube or to another spatial shape. In our approach, we consider the general case with a nonlinear measurement equation. The main contributions of the work is in the derived likelihood function based on a parameterised shape and in the developed SMC filter for extended objects. Then we propagate this spatial measurement uncertainty through the Bayesian estimation framework.

In 2005, Gilholm and Salmond [GS05] developed a spatial distribution model for tracking extended objects in clutter, where the number of observations from the target is assumed to be Poisson distributed. Based on this approach Poisson likelihood models for group and extended object tracking were developed [CG07].

The rest of this paper is organised as follows. Section 2 introduces the SMC framework in the case of EOT. Section 3 gives the measurement likelihood in the presence of clutter. Then performance evaluation scenario is presented as well as conclusions based on it in Sections 4 and 5, respectively.

2 Extended Object Tracking Within the SMC Framework

This work considers the state estimation problem for extended objects with clutter noise. Such objects usually give rise to a set of measurements. The system dynamics and sensor can be described using the equations

$$\boldsymbol{x}_k = f(\boldsymbol{x}_{k-1}, \boldsymbol{\eta}_{k-1}), \tag{1}$$

$$\boldsymbol{z}_k = h(\boldsymbol{x}_k, \boldsymbol{w}_k), \tag{2}$$

where $\boldsymbol{x}_k = \left(\boldsymbol{X}_k^T, \boldsymbol{\Theta}_k^T\right)^T \in \mathbb{R}^{n_x}$, with $(\cdot)^T$ being the transpose operator, is the unknown system state vector at time step k, $k = 1, 2, ..., K$, where K is the maximum number of time steps. The vector $\boldsymbol{x}_k$ consists of the object kinematic state vector $\boldsymbol{X}_k$ and object extent, characterised by the parameter vector $\boldsymbol{\Theta}_k \in \mathbb{R}^{n_\Theta}$; $f(.)$ and $h(.)$ are respectively the system and the measurement functions, nonlinear in general; $\boldsymbol{z}_k \in \mathbb{R}^{n_z}$ is the measurement vector and $\boldsymbol{\eta}_k = (\boldsymbol{\eta}_{s,k}^T \ \ \boldsymbol{\eta}_{p,k}^T)^T$ and $\boldsymbol{w}_k$ are the system (kinematic state and parameters) and measurement noises, respectively.

Using the classical Bayesian theory, the ChapmanKolmogorov equation and the Particle Filter (PF) approach [AMGC02] the posterior pdf $p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k})$ can be approximated with a set of particles $\boldsymbol{x}_k^{(i)}, \ i = 1, \ldots, N$ and their corresponding weights $w_k^{(i)}$.

A *resampling* procedure introduces variety in the particles by eliminating those with small weights and replicating the particles with larger weights.

3 Measurement Likelihood in the Presence of Clutter

The aim of this paper is to derive general likelihood calculation procedures based on Monte Carlo methods without a restriction on the type of set of interest or the type of cluttered measurements available. The main idea of this paper is to introduce a sampling step for regions of interest in the extended object. This sampling aims to represents the probability of a given point, in the state space, to be the origin of a measurement. This section describes briefly the newly proposed method, details can be found in the references herein. As an illustration, the case of an extended object with circular form is considered.

Without loss of generality, assume that there is only one static sensor described by its state vector $\boldsymbol{x}_{s,k}$. Assume that at each measurement time k the extended object generates a matrix $\boldsymbol{Z}_k = \{\boldsymbol{z}_k^1, \ldots, \boldsymbol{z}_k^m\} \in \mathbb{R}^{n_z \times M_k}$ of $M_k = M_{T,k} + M_{C,k}$ measurement vectors. The number of measurements $M_{T,k}$ originating from the visible border of the source is considered Poisson-distributed random variable with mean value of λ_T, or $M_{T,k} \sim Poisson(\lambda_T)$. Similarly, the number of clutter measurements is $M_{C,k} \sim Poisson(\lambda_C)$, where λ_C is the mean value of the clutter measurements. The clutter measurements are modeled according to [DM01]. All of the measurements are assumed to be conditionally independent, i.e.

$$p(\boldsymbol{Z}_k|\boldsymbol{x}_k) = \prod_{j=1}^{M_k} p(\boldsymbol{z}_k^j|\boldsymbol{x}_k). \tag{3}$$

If at time step $k-1$ the posterior pdf $p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{k-1}^j)$ is known, then one can express the prior $p(\boldsymbol{x}_k|\boldsymbol{z}_{k-1}^j)$ via the Chapman-Kolmogorov equation. The nearly constant velocity model [LJ03,PMGA11] is used to model the motion of the centre of the region surrounding the extended target.

3.1 Observation Model

The measurement vector is $\boldsymbol{z}_k^j = (r_k^j, \beta_k^j)^T$, where r_k^j is the range and β_k^j is the bearing of the measurement point j. The equation for the measurements originating from the target has the form: $\boldsymbol{z}_k^j = h(\boldsymbol{x}_k) + \boldsymbol{w}_k^j$, where h is the nonlinear function $h(\boldsymbol{x}_k) = \left(\sqrt{x_k^{j2} + y_k^{j2}}, \tan^{-1}\frac{y_k^j}{x_k^j}\right)$, x_k^j and y_k^j denote the Cartesian coordinates of the actual point of the source from where the measurement emanates in the case of two dimensional space. The measurement noise $\boldsymbol{w}_k^j$ is supposed to be Gaussian, with a known covariance matrix $\boldsymbol{R} = diag(\sigma_r^2, \sigma_\beta^2)$. The clutter measurements are considered uniformly distributed within the visible area of the sensor.

3.2 Introduction of the Notion of the Visible Surface

The measurement likelihood $p(\boldsymbol{z}_k^j|\boldsymbol{x}_k)$ of the extended target can be represented with the relation

$$p(\boldsymbol{z}_k^j|\boldsymbol{x}_k) = \int\limits_{\mathbb{R}^{n_v}} p(\boldsymbol{z}_k^j|\boldsymbol{V}_k)p(\boldsymbol{V}_k|\boldsymbol{x}_k)d\boldsymbol{V}_k, \tag{4}$$

where $\boldsymbol{V}_k \in \mathbb{R}^{n_v}$ denotes a source of measurement in the state space. In practice, these visible sources $\boldsymbol{V}_k$ depend on the object position, nature and parameters $\boldsymbol{x}_k, \lambda_T$ or scatter characteristics λ_C and on the sensor state (e.g., the sensor position and angle of view). The pdf $p(\boldsymbol{V}_k|\boldsymbol{x}_k)$ represents the probability of a point in the state space to be a source of measurement given the extended object $\boldsymbol{x}_k$. The surface of the target with state $\boldsymbol{x}_k$ visible from the sensor with state $\boldsymbol{x}_{s,k}$ is denoted by $\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})$.

We assume that the sources of the true measurements at time step k, given the target state and the sensor prior state are uniformly distributed along the region $\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})$, visible from the sensor $x_{s,k}$, i.e.

$$p(\boldsymbol{V}_k|\boldsymbol{x}_k) = p(\boldsymbol{V}_k|\boldsymbol{x}_k, \boldsymbol{x}_{s,k}) = \mathcal{U}_{\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})}(\boldsymbol{V}_k), \tag{5}$$

where $\mathcal{U}_{\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})}(\cdot)$ represents the uniform pdf with the support $\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})$. Inserting (5) into (4) gives

$$p(\boldsymbol{z}_k^j|\boldsymbol{x}_k) = \int\limits_{\mathbb{R}^{n_v}} p(\boldsymbol{z}_k^j|\boldsymbol{V}_k)\mathcal{U}_{\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})}(\boldsymbol{V}_k)d\boldsymbol{V}_k = \frac{1}{||\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})||} \int\limits_{\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})} p(\boldsymbol{z}_k^j|\boldsymbol{V}_k)d\boldsymbol{V}_k, \tag{6}$$

where $||\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})||$ denotes some measure of the region $\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})$. Unfortunately that integral (6) is difficult to calculate. Gating is applied to the spatial area of the measurement sources. The gating region is defined by an angular field ($\beta_1 < \beta < \beta_2$) and minimum and maximum distance ($d_1 < d < d_2$) around the predicted object center with respect to sensor position. An example of such gating as well as some notations are shown for object with circular extent in Fig. 1.

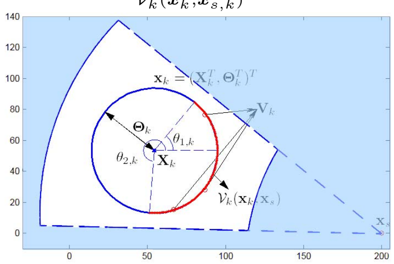


Fig. 1: Notations and definitions in the context of an example with circular extent

3.3 Parametrisation of the Visible Border

The calculation of the likelihood (4) can be performed using a Monte Carlo method in the following way. After the prediction step, a set of weighted particles $\{(\boldsymbol{x}_{k|k-1}^{(i)}, w_{k|k-1}^{(i)})\}_{i=1}^N$ is available (recall that each of the N particles can be seen as an extended object hypothesis). First, the visible border $\mathcal{V}_k(\boldsymbol{x}_{k|k-1}^{(i)}, \boldsymbol{x}_{s,k})$ is determined. Then for each particle $\boldsymbol{x}_{k|k-1}^{(i)}$

the likelihood function $p(\boldsymbol{V}_k|\boldsymbol{x}_{k|k-1}^{(i)}, \boldsymbol{z}_k^j)$ is defined from the support $\mathcal{V}_k(\boldsymbol{x}_{k|k-1}^{(i)}, \boldsymbol{x}_{s,k})$, taking into account the angular information of the measurement $\boldsymbol{z}_k^j$. The measurements $\{\boldsymbol{z}_k^j\}_{j=1}^M$ are generated according to $\mathcal{N}(\boldsymbol{z}_k^j, h(\boldsymbol{V}_k^{(i,\ell)}), \boldsymbol{R})$. Then sampling Q visible points from each of M_k measurements gives us a total of $S = QM_k$ samples per particle $\{\boldsymbol{V}_k^{(i,\ell)}\}_{\ell=1}^S$, for each particle i:

$$\boldsymbol{V}_k^{(i,j)} \sim \begin{pmatrix} r_{v,k}^{(i,j)} \\ \theta_{v,k}^{(i,j)} \end{pmatrix} = \begin{pmatrix} \mathcal{N}(r_{v,k}^{(i,j)}; r_k^{(i)}, \tilde{\sigma}_r^2) \\ \mathcal{N}(\theta_{v,k}^{(i,j)}; \theta_k^{(i,j)}, \tilde{\sigma}_\theta{}^2) \end{pmatrix}, \tag{7}$$

where $\tilde{\sigma}_r$ and $\tilde{\sigma}_\theta$ are the standard deviations for the radius and angle, respectively, chosen for generating the samples.

Once the points $\{\boldsymbol{V}_k^{(i,\ell)}\}_{\ell=1}^S$ are available for each particle $\boldsymbol{x}_{k|k-1}^{(i)}$, we can approximate the likelihood (see the equation (4)) according to

$$p(\boldsymbol{z}_k^j|\boldsymbol{x}_{k|k-1}^{(i)}) = \int_{\mathbb{R}^{n_x}} p(\boldsymbol{z}_k^j|\boldsymbol{V}_k)p(\boldsymbol{V}_k, \boldsymbol{x}_{k|k-1}^{(i)})d\boldsymbol{V}_k, = \sum_{\ell=1}^{M} p(\boldsymbol{z}_k^j|\boldsymbol{V}_k^{(i,\ell)})p(\boldsymbol{V}_k^{(i,\ell)}|\boldsymbol{x}_{k|k-1}^{(i)}). \tag{8}$$

The term $p(\boldsymbol{z}_k^j|\boldsymbol{V}_k^{(i,\ell)})$ in (8) can be easily calculated in a classical way depending on the problem (for example using a Gaussian likelihood). The term $p(\boldsymbol{V}_k^{(i,\ell)}|\boldsymbol{x}_{k|k-1}^{(i)})$ depends very much on the visible set $\mathcal{V}_k(\boldsymbol{x}_{k|k-1}^{(i)}, \boldsymbol{x}_{s,k})$. It can be seen as $p(\boldsymbol{V}_k^{(i,\ell)}|\boldsymbol{x}_{k|k-1}^{(i)}) = p(\boldsymbol{V}_k^{(i,\ell)} \in \mathcal{V}_k(\boldsymbol{x}_{k|k-1}^{(i)}, \boldsymbol{x}_{s,k}))$. Here we will give an example for the calculation of the term $p(\boldsymbol{V}_k^{(i,\ell)}|\boldsymbol{x}_{k|k-1}^{(i)})$ over a circle. Let us denote with $x_k^{(i,\ell)}$ and $y_k^{(i,\ell)}$ the coordinates of $\boldsymbol{V}_k^{(i,\ell)}$, i.e. $\boldsymbol{V}_k^{(i,\ell)} = (x_k^{(i,\ell)}, y_k^{(i,\ell)})^T$. For circular object $\mathcal{V}_k(\boldsymbol{x}_{k|k-1}^{(i)}, \boldsymbol{x}_{s,k})$ can be given as

$$\mathcal{V}_k(\boldsymbol{x}_{k|k-1}^{(i)}, \boldsymbol{x}_{s,k}) = \{(x_{c,k|k-1}^{(i)} + r_{k|k-1}^{(i)}\cos(\theta)y_{c,k|k-1}^{(i)} + r_{k|k-1}^{(i)}\sin(\theta)), \theta \in [\theta_{1,k|k-1}^{(i)}, \theta_{2,k|k-1}^{(i)}]\}.$$

A simple expression of $p(\boldsymbol{V}_k^{(i,\ell)}|\boldsymbol{x}_{k|k-1}^{(i)})$ can be

$$p(\boldsymbol{V}_k^{(i,\ell)}|\boldsymbol{x}_{k|k-1}^{(i)}) = \mathcal{N}(\sqrt{(x_k^{(i,\ell)} - x_{c,k}^{(i)})^2 + (y_k^{(i,\ell)} - y_{c,k}^{(i)})^2}, r_{k|k-1}^{(i)}, \sigma_v^2) \tag{9}$$

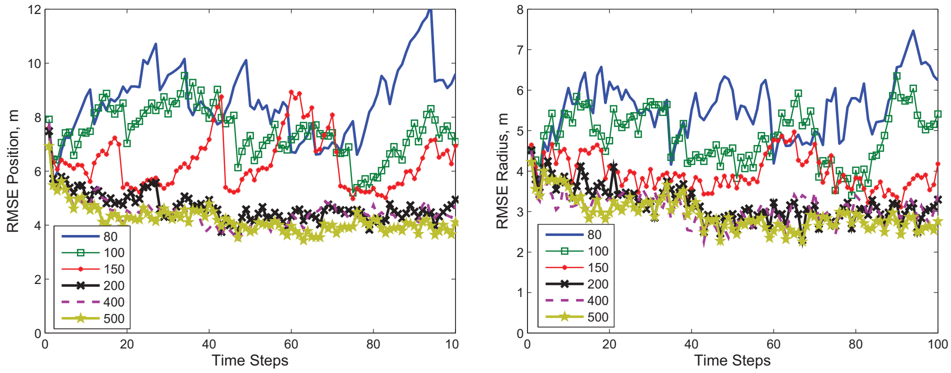
where σ_v is the standard deviation for a Gaussian likelihood applied to the distance from $\boldsymbol{V}_k^{(i,\ell)}$ to the center of the particle $(x_{c,k}^{(i)}, y_{c,k}^{(i)})$. Similarly to [GS05], in the presence of clutter the likelihood function of the PF can be calculated from the following equation

$$P(\boldsymbol{Z}_k|\boldsymbol{x}_k^{(i)}) = \prod_{j=1}^{M} \left(1 + \frac{\lambda_T}{\rho} p(\boldsymbol{z}_k^j|\boldsymbol{x}_{k|k-1}^{(i)})\right), \tag{10}$$

where $\rho = \lambda_C/A$ is the uniform clutter density and A is the area visible from the sensor.

4 Performance Evaluation

An example, similar to the one presented in [PMGA11] is considered with added clutter noise. Simulations are performed for 100 time steps each repeated for 500 iterations in the case of tracking of object with circular extent and cluttered measurements consisting of range and bearing. Scenarios with extended object particles from 80 to 500 are considered. Please note that the actual number of samples is much higher and depends on the number of measurements. For each of particles 5 Metropolis-Hastings proposals are generated. The initial position of the object is biased by Gaussian random noise with standard deviation



(a) RMSE of the Position (b) RMSE of the Radius

Fig. 2: Comparison of the results for number of particles varying from 80 to 500

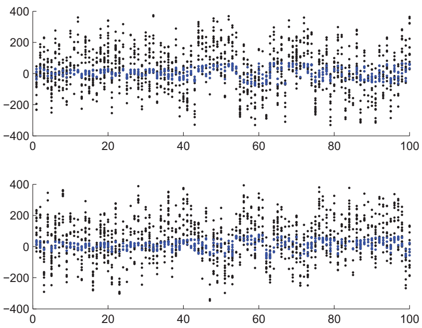


Fig. 3: Clutter/real measurements xy-plot

N particles	Success rate, %
80	68.6
100	69.2
150	73.4
200	80.4
400	83.2
500	79.6

Table 1 Success rate

of 10 m for both coordinates. The initial extent is altered by Gaussian noise with standard deviation of 5 m in radius, and the initial velocity is generated with added Gaussian noise with standard deviation of 1.5 m/s for each of the coordinates.

The number of both the target and the clutter measurements generated in each step is assumed to be Poisson distributed, i.e. $M_{T,k} \sim Poisson(\lambda_T)$, where $\lambda_T = 5$ and $M_{C,k} \sim Poisson(\lambda_C)$, where $\lambda_C = 13$. For each measurement and each hypothesis one random sample Q is generated according to (7). The radius of visibility of the sensor is assumed to be 200 m, therefore $\rho = 0.0001$. The standard deviations for the bearing and for the radius of these samples are respectively $\sigma_\beta = 3°$ and $\sigma_r = 1\,m$. The sensor line of sight for simplicity is determined by the angles $\alpha_{1,k} = 0$ and $\alpha_{2,k} = 2\pi$.

The real measurements are assumed to originate from random locations of the visible frontier of the extent where the angular position is uniformly distributed over the visible arc and the range has additive random Gaussian noise with standard deviation of $2\,m$. The clutter measurements are uniformly distributed in the visible area of the sensor.

The change of the size of the extent in the space-state evolution model is generated by adding/substracting (corresponding to extension/shrinkage) the absolute value of a Gaussian random variable with zero mean and std $2\,m$. The actual target trajectory is generated based on (1) with noise covariance equal to zero. The change of the size of the extent when generating the extended object particles is modeled a random walk using normally distributed random variable with zero mean and standard deviation $4\,m$. The trajectory prediction in the filter is performed with standard deviation for the components of the system dynamic noise $\sigma_x = 1\, m/s^2$ and $\sigma_y = 1\, m/s^2$, respectively. The results from the simulations are presented in Figs. 2 - 3 and Table 1. The success rate refers to the percentage of successfully tracked trajectories.

5 Conclusions

This paper presents an approach for coping with clutter noise when tracking extended objects. It is shown that accurate results are obtained with at least 200 particles for the considered testing example.

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References

[AMGC02] M. Arulampalam, S. Maskell, N. Gordon und T. Clapp. A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking. *IEEE Trans. on Signal Proc.*, 50(2):174–188, 2002.

[BFF+10] M. Baum, M. Feldmann, D. Fränken, U. D. Hanebeck und W. Koch. Extended Object and Group Tracking: A Comparison of Random Matrices and Random Hypersurface Models. In *GI Jahrestagung (2)*, Seiten 904–906, 2010.

[BH09a] M. Baum und U. D. Hanebeck. Extended Object Tracking based on Combined Set-Theoretic and Stochastic Fusion. In *Proc. of the Internat. Conf. on Inf. Fusion*, 2009.

[BH09b] M. Baum und U.D. Hanebeck. Random Hypersurface Models for extended object tracking. In *Proc. of the IEEE International Symp. on Signal Processing and Information Technology (ISSPIT)*, Seiten 178 –183, 2009.

[CG07] D. Clark und S. Godsill. Group Target Tracking with the Gaussian Mixture Probability Density Filter. In *Proc. of the 3rd International Conf. on Intelligent Sensors, Sensor Networks and Information Processing*, 2007.

[DM01] J. Dezert und C. Musso. An Efficient Method for Generating Points Uniformly Distributed in Hyperellipsoids. In *Proc. of Workshop on Estimation, Tracking and Fusion: A Tribute to Yaakov Bar-Shalom, Naval Postgraduate School*. Monterey, CA, 2001.

[GS05] K. Gilholm und D. Salmond. Spatial Distribution Model for Tracking Extended Objects. *IEE Proc.-Radar, Sonar Navig.*, 152(5):364–371, 2005.

[KF09] W. Koch und M. Feldmann. Cluster tracking under kinematical constraints using random matrices. *Robotics and Autonomous Systems*, 57(3):296 – 309, 2009.

[Koc08] J.W. Koch. Bayesian approach to extended object and cluster tracking using random matrices. *IEEE Trans. on Aerospace and Electronic Syst.*, 44(3):1042–1059, July 2008.

[LJ03] X. R. Li und V. Jilkov. A Survey of Maneuveuvering Target Tracking. Part I: Dynamic Models. *IEEE Trans. on Aerosp. and Electr. Systems*, 39(4):1333–1364, 2003.

[NKPH10] B. Noack, V. Klumpp, N. Petkov und U. Hannebeck. Bounding Linearisation Errors with Sets of Densities in Approximate Kalman Filtering. In *Proc. of the 13th International Conf. on Information Fusion*, Edinburgh, UK, 2010. ISIF.

[PMGA11] N. Petrov, L. Mihaylova, A. Gning und D. Angelova. A Novel Sequential Monte Carlo Approach for Extended Object Tracking Based on Border Parameterisation. In *Proc. of the 14th International Conf. on Information Fusion*, Chicago, USA, 7 2011.

[VIG05] J. Vermaak, N. Ikoma und S. Godsill. Sequential Monte Carlo Framework for Extended Object Tracking. *IEE Proc.-Radar, Sonar Navig.*, 152(5):353–363, 2005.